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15/ENG05/017
MECHATRONICS
ENG 381

1 $d^2y/d\theta + 4dy/d\theta + 5y = 6\sin\theta$

Assume homogenity

$d^{2y}/d\theta^2 + 4dy/d\theta + 5y = 0$

$K^2 + 4K + 5 = 0$

$a = 1, b = 4, c = 5$ using $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$K = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)} \Rightarrow \frac{-4 \pm \sqrt{4}}{2}$

$K = \frac{-4 \pm 2y}{2}$ $\therefore K = -2 \pm y$

If $y = e^{-2\theta}(A\cos\theta + B\sin\theta)$

$y = (C\cos\theta + D\sin\theta)$

$dy/d\theta = -C\sin\theta + D\cos\theta$

$-C\cos\theta - D\sin\theta + 4[-C\sin\theta + D\cos\theta] + 5[C\cos\theta + D\sin\theta] = 6\sin\theta$

$-C\cos\theta - D\sin\theta - 4C\sin\theta + 4D\cos\theta + 5C\cos\theta + 5D\sin\theta = 6\sin\theta$

$-C + 4D + 5C = 0$

$-D + 4C + 5D = 6$

$4C + 4D = 0$ ——— (1)

$+\ -4C + 4D = 6$ ——— (2)

$8D = 6$

$D = 6/8 = 3/4$

Substitute $D = 3/4$ in equation (2)

$-4C + 4(3/4) = 6$

$-4C + 3 = 6$

$-4C = 3$ $\therefore C = -3/4$

$y = -3/4\cos\theta + 3/4\sin\theta$

$y = e^{-2\theta}(A\cos\theta + B\sin\theta) - 3/4\cos\theta + 3/4\sin\theta$

$y = e^{-2\theta}(A\cos\theta + B\sin\theta) + 3/4(\sin\theta - \cos\theta)$

at steady state

$dy/dx = 0$ and $\theta = \alpha$

$$y = e^{-2\theta}(A\cos\theta + B\sin\theta) + \frac{3}{4}(\sin\theta - \cos\theta)$$

$$dy/d\theta = e^{-2\theta}(B\cos\theta - A\sin\theta) - 2e^{-2\theta}(A\cos\theta + B\sin\theta) + \frac{3}{4}(\sin\theta - \cos\theta$$

$$dy/d\theta = e^{-2\alpha}(B\cos\alpha - A\sin\alpha) - 2e^{-2\alpha}(A\cos\alpha + B\sin\alpha) + \frac{3}{4}(\sin\alpha - \cos\alpha$$

$$dy/d\theta = 3/4(\sin\alpha - \cos\alpha)$$

$$dy/d\theta = 3/4(\sin\theta - \cos\theta)$$

2

$$EI\, d^2y/dx^2 = w/2 (L-x)^2$$

$$d^2y/dx^2 = \frac{Bo}{2EI}(L-x)^2$$

$$y'' = \frac{w}{2EI}(L-x)^2$$

$$K^2 = \frac{w}{2EI}(L-x)^2 \quad \therefore K = \pm\sqrt{\frac{w(L-x)^2}{2EI}}$$

$$y = C_1\cosh\left(\sqrt{\frac{w(L-x)^2}{2EI}}\right) + C_2\sinh\left(\sqrt{\frac{w(L-x)^2}{2EI}}\right)$$

$$y = Rx^2 + Sx^3 + Tx^4$$

$$dy/dx = 2Rx + 3Sx^2 + 4Tx^3$$

$$d^2y/dx^2 = 2R + 6Sx + 12Tx^2$$

$$EI[2R + 6Sx + 12Tx^2] = w/2 (L-x)^2$$

$$2REI + 6SxEI + 12Tx^2EI = w/2[L^2 - 2Lx + x^2]$$

$$4REI + 12SxEI + 24Tx^2EI = wL^2 - 2wLx + wx^2$$

$$24TEI = w$$

$$T = \frac{w}{24EI}$$

$$12SEI = -2wL$$

$$S = \frac{-2wL}{24EI}$$

$$Y = \left[\frac{wL^2}{4EI}\right]x^2 - \left[\frac{wL}{6EI}\right]x^2 + \left[\frac{w}{24EI}\right]x^4$$

$$y = \frac{6wL^2x^2 - 4wLx^3 + wx^4}{24EI}$$

$$y = \frac{6wL^2x^2 - 4wLx^3 + wx^4}{24EI}$$

$P_I \Rightarrow y = \frac{w}{24EI}\left[6L^2x^2 - 4Lx^3 + x^4\right]$

$y = C_1 \cosh\left(\sqrt{\frac{w(L-x)^2}{2EI}}\right) + C_2 \sinh\left[\sqrt{\frac{w(L-x)^2}{2EI}}\right] + \frac{w}{24EI}\left[6L^2x^2 - 4Lx^2 + x^4\right]$

at $x=0, y=0 \quad dy/dx = 0$

$0 = C_1 \cosh\left(\sqrt{\frac{w(L-0)^2}{2EI}}\right) + C_2 \sinh\left(\sqrt{\frac{w(L-0)^2}{2EI}}\right) + \frac{w}{24EI}\left[6L^2(0)^2 - 4L(0)^2 - (0)^2\right]$

$0 = C_1 \cosh\left(\sqrt{\frac{wL^2}{2EI}}\right) + C_2 \sinh\left(\sqrt{\frac{wL^2}{2EI}}\right) + 0$

When $x - L$

$y = w/24EL\left[6L^4 - 4L^4 + L^4\right]$

$y = \frac{w}{24EI}\left[3L^4\right]$

$y = \frac{wL^4}{8EI}$

graph1 Sheet1