

Uchechi Benjamin

Petroleum Engineering

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ENG 281

1a Evaluate $\lim_{x \rightarrow \pi/2} \frac{x^2 - \pi/4 \sin(\cos x)}{x - \pi/2}$

$$= \frac{(\pi/2)^2 - \pi/4 \sin(\cos(\pi/4))}{\pi/2 - \pi/2} = \frac{0}{0} \text{ Undefined}$$

$$\lim_{x \rightarrow \pi/2} \frac{(2x-0)(\sin(-\sin x) + \cos x(\cos x))}{1-0}$$
$$= 2 \left(\frac{\pi}{2} \right) \left[-\sin^2\left(\frac{\pi}{2}\right) + \cos^2\left(\frac{\pi}{2}\right) \right]$$

$$= \pi(-1+0)$$

$$= -\pi$$

$$\lim_{x \rightarrow \pi/2} \frac{x^2 - \pi/4 \sin(\cos x)}{x - \pi/2} = -\pi$$

b) $\lim_{x \rightarrow \pi/2} \ln \frac{\exp(3x^2 + 2x - 1)}{x + 1}$

$$\lim_{x \rightarrow \pi/2} \frac{(3x-1)(x+1)}{(x+1)}$$

$$\lim_{x \rightarrow \pi/2} (3x-1) = 3\left(\frac{\pi}{2}\right) - 1$$
$$= 3\frac{\pi}{2} - 1$$

$$\lim_{x \rightarrow \pi/2} \ln \left[\frac{\exp(3x^2 + 2x - 1)}{x + 1} \right] = \frac{3\pi - 2}{2}$$

c) $\lim_{x \rightarrow 2+\sqrt{3}} \cos \left[\frac{\sin^{-1}(x-2)}{(x-\sqrt{3})} \right]$

$$\cos \left[\frac{\sin^{-1}(2+\sqrt{3}-2)}{(2+\sqrt{3}-\sqrt{3})} \right]$$

$$\cos \left[\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \right]$$

$$\lim_{x \rightarrow 2+\sqrt{3}} \cos \left[\frac{\sin^{-1}(x-2)}{(x-\sqrt{3})} \right] = \cos 60 = \frac{1}{2}$$

$$d) \lim_{x \rightarrow 4} \left[\frac{x^2 - 8x + 16}{x^2 - 5x + 4} \right]$$

$$= \frac{4^2 - 8(4) + 16}{4^2 - 5(4) + 4} = \frac{0}{0} \text{ undefined}$$

$$\lim_{x \rightarrow 4} \frac{2x - 8}{2x - 5}$$

$$= \frac{2(4) - 8}{2(4) - 5} = \frac{0}{3} = 0$$

$$\lim_{x \rightarrow 4} \left[\frac{x^2 - 8x + 16}{x^2 - 5x + 4} \right] = 0$$

$$b) \frac{2}{1^2}$$

$$U_n$$

$$\lim_{n \rightarrow \infty}$$

$$2a) \frac{2}{2 \times 3} + \frac{2}{3 \times 4} + \frac{2}{4 \times 5} + \frac{2}{5 \times 6} + \dots$$

$$U_n = \frac{2}{(n+1) \times (n+2)}, \quad U_{n+1} = \frac{2}{(n+1+1)(n+2+1)} = \frac{2}{(n+2)(n+3)}$$

$$\lim_{n \rightarrow \infty} \frac{U_{n+1}}{U_n} = \frac{2}{(n+2)(n+3)} \times \frac{(n+1)(n+2)}{2}$$

$$= \frac{n^2 + 3n + 2}{n^2 + 5n + 6}$$

divide through by the highest power of n

$$\lim_{n \rightarrow \infty} \frac{U_{n+1}}{U_n} = \frac{\frac{n^2}{n^2} + \frac{3n}{n^2} + \frac{2}{n^2}}{\frac{n^2}{n^2} + \frac{5n}{n^2} + \frac{6}{n^2}}$$

$$= \frac{1 + 0 + 0}{1 + 0 + 0} = \frac{1}{1} = 1$$

It maybe either convergent or divergent using test 1 on U_n

$$U_n = \frac{2}{n^2 + 3n + 2}$$

$$= \frac{2/n^2}{n^2/n^2 + 3n/n^2 + 2/n^2} = \frac{0}{1 + 0 + 0}$$

$$= 0 \text{ converge}$$

$$b) \quad \frac{2}{1^2} + \frac{2^2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \dots$$

$$U_n = \frac{2}{n^2}, \quad U_{n+1} = \frac{2}{(n+1)^2}$$

$$\lim_{n \rightarrow \infty} \frac{U_{n+1}}{U_n} = \frac{2}{(n+1)^2} \times \frac{n^2}{2}$$

$$= \frac{n^2}{n^2 + 2n + 1}$$

divide by the highest power of n

$$= \frac{\frac{n^2}{n^2}}{\frac{n^2}{n^2} + \frac{2n}{n^2} + \frac{1}{n^2}} = \frac{1}{1 + 0 + 0} = \frac{1}{1} \quad \text{may converge or diverge}$$

Using test 1 on U_n

$$U_n = \frac{2}{n^2}$$

$$\frac{\frac{2}{n^2}}{\frac{2}{n^2}} = \frac{0}{1} = 0 \quad \text{converge}$$

$$c) \quad U_n = \frac{1 + 2n^2}{1 + n^2}$$

$$\lim_{n \rightarrow \infty} U_n = \frac{\frac{1}{n^2} + \frac{2n^2}{n^2}}{\frac{1}{n^2} + \frac{n^2}{n^2}}$$

$$= \frac{0 + 2}{0 + 1} = \frac{2}{1} = 2 \quad \text{diverge}$$

$$3) \quad \frac{x}{27} + \frac{x^2}{125} + \dots + \frac{x^n}{(2n+1)^3}$$

$$U_n = \frac{x^n}{(2n+1)^3}, \quad U_{n+1} = \frac{x^{n+1}}{(2(n+1)+1)^3} = \frac{x^{n+1}}{(2n+3)^3}$$

$$\lim_{n \rightarrow \infty} \frac{U_{n+1}}{U_n} = \frac{x^{n+1}}{(2n+3)^3} \times \frac{(2n+1)^3}{x^n}$$

$$= \frac{(x)(2n+1)^3}{(2n+3)^3}$$

$$= (8n^3 + 12n^2 + 6n + 1)x$$

$$8n^3 + 18n^2 + 54n + 27$$

$$= \frac{8xn^3 + 12xn^2 + 6xn + x}{8n^3 + 18n^2 + 54n + 27}$$

$$8n^3 + 18n^2 + 54n + 27$$

divide by highest power of n.

$$= \frac{\frac{8xn^3}{n^3} + \frac{12xn^2}{n^3} + \frac{6xn}{n^3} + \frac{x}{n^3}}{\frac{8n^3}{n^3} + \frac{18n^2}{n^3} + \frac{54n}{n^3} + \frac{27}{n^3}}$$

$$= \frac{8x}{8}$$

$$\frac{8x}{8} < 1$$

$$8x < 8$$

$$x < 1$$

$$4) \lim_{x \rightarrow 0} \left[\frac{\sin x - \cos x}{x^3} \right]$$

$$\frac{\sin 0 - \cos 0}{0^3} = \frac{0 - 1}{0} \text{ Undefined}$$

$$\lim_{x \rightarrow 0} \frac{\cos 0 + \sin 0}{3(0)^2} = \frac{1 + 0}{0} \text{ Undefined}$$

$$\lim_{x \rightarrow 0} \frac{-\sin x + \cos 2x}{6x} = \frac{-\sin 0 + \cos 0}{6(0)} = \frac{-0 + 1}{0} \text{ Undefined}$$

$$\lim_{x \rightarrow 0} \frac{-\cos 2x - \sin x}{6} = \frac{-\cos 0 - \sin 0}{6} = \frac{-1 - 0}{6} = -\frac{1}{6}$$