The parametric equations of a curve are as given in Equations (1) & (2)

$x = \cos t + t\sin t$ --- (1)

$y = \sin t - t\cos t$ --- (2)

In terms of t, determine

(i) an expression for the radius of curvature (ρ) and

(ii) expressions for the coordinates (h, k) of the centre of curvature

Solution

1. $x = \cos t + t\sin t$

$y = \sin t - t\cos t$

$\frac{dx}{dt} = -\sin t + t\cos t + \sin t = t\cos t$

$t\sin t$

using product rule

$u = t \quad v = \sin t$

$\frac{du}{dt} = 1 \quad \frac{dv}{dt} = \cos t$

$u\frac{dv}{dt} + v\frac{du}{dt}$

$= t(\cos t) + \sin t(1)$

$= t\cos t + \sin t$

$\frac{dy}{dt} = \cos t + t\sin t - \cos t = t\sin t$

using product rule

$u = -t \quad v = \cos t$

$\frac{du}{dt} = -1 \quad \frac{dv}{dt} = -\sin t$

$V\frac{dy}{dt} + V\frac{dy}{dt}$

$= -t(-\sin t) + \cos t(-1)$

$= t\sin t - \cos t$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dx}{dt}$

$= \frac{t\sin t}{t\cos t} \quad \frac{\cancel{t}\sin t}{\cancel{t}\cos t}$

$= \tan t$

$\frac{d^2y}{dx^2} = \sec^2 t$

$1.\ R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left(\frac{d^2y}{dx^2}\right)}$

$= \frac{[1 + (\tan t)]^2}{[\sec^2 t]}$

$h = x - R\sin\theta$

$k = y + R\cos t$

$\tan t = \frac{dy}{dx}$

$h = [\cos t + t\sin t] - \left[\frac{[1 + [\tan t]^2]^{3/2}}{[\sec^2 t]}\right] \sin [\tan^{-1}[\tan t]]$

$k = [\sin t - t\cos t] + \left[\frac{[1 + [\tan t]^2]^{3/2}}{[\sec^2 t]}\right] \cos [\tan^{-1}[\tan t]]$

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