Assignment 2

Oong-bye Abum
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(1) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 6\sin\theta$

C.F

$m^2 + 4m + 5 = 0$

$m = -2 \pm j$

$y = e^{-2\theta}(A\cos\theta + B\sin\theta)$

Assumed P.I.

$y = C\cos\theta + \Delta\sin\theta$

$\frac{dy}{dx} = -C\sin\theta + \Delta\cos\theta$

$\frac{d^2y}{dx^2} = -C\cos\theta - \Delta\sin\theta$

$-C\cos\theta - \Delta\sin\theta - 4C\sin\theta + 4\Delta\cos\theta$
$+ 5C\cos\theta + 5\Delta\sin\theta = 6\sin\theta$

$\sin\theta = -\Delta - 4C + 5\Delta = 6$

$\cos\theta = -C + 4\Delta + 5C = 0$

$4\Delta - 4C = 6$ — (i)
$4C + 4\Delta = 0$ — (ii)

$4\Delta - 4C = 6$
$4\Delta + 4C = 0$

$-8C = 6$

$C = \frac{-6}{8} = \frac{-3}{4}$

$C = -3/4$

From equ (ii)

$4(-3/4) + 4\Delta = 0$

$\frac{-3}{-3} = \frac{+4\Delta}{+3}$

$\Delta = 3/4$

Assumed PI

$y = -3/4\cos\theta + \frac{3}{4}\sin\theta$

$y = 3/4\sin\theta - 3/4\cos\theta$

G.S

$y = e^{-2\theta}(A\cos\theta + B\sin\theta) + 3/4\sin\theta$

$y = 3/4(\sin\theta - \cos\theta)$

for $\theta = 0° \text{ to } 270°$

$y = 3/4(\sin\theta - \cos\theta)$

$\frac{dy}{d\theta} = 3/4\sin\theta - 3/4\cos\theta$

$\frac{dy}{d\theta} = 3/4\cos\theta + 3/4\sin\theta$

at steady state

$\frac{dy}{d\theta} = 0$ and $\theta = \infty$

$0 = 3/4(\cos\theta + \sin\theta)$

$-\cos\theta = 1\sin\theta$

divide both side by

$\frac{-\cos\theta}{\cos\theta} = \frac{\sin\theta}{\cos\theta}$

$\tan\theta = -1$

$\theta = \tan^{-1}(-1)$

$\theta = 45$

$\theta = \underline{315°}$

(2) $EI\frac{d^2y}{dx^2} = \frac{wl}{2}(l-x)^2$

C.F

Auxiliary equ

$m^2 = 0$

$m = \sqrt{0}$

$m = \pm 0$

$y = e^0(A + Bx)$

$y = A + Bx$

$y = Cx^2 + Dx^3 + Fx^4$

$\frac{dy}{dx} = 2Cx + 3Dx^2 + 4Fx^3$

$\frac{d^2y}{dx^2} = 2C + 6Dx + 12Fx^2$

Putting eqn 3 into the original equation

$EI(2C + 6Dx + 12Fx^2) = \frac{w}{2}(l-x)^2$

$2CEI + 6DEIx + 12EFIx^2 = \frac{w}{2}(l-x)^2$

Comparing coefficient

x^2: $24EFI = 2w$

$F = \frac{w}{24EI}$

x': $12DEI = -2wl$

$D = \frac{-2wl}{12EI}$

Constant: $4CEI = wl^2$

$C = \frac{wl^2}{4EI}$

Putting back into the original equation

$y = \frac{wl^2}{4EI}x^2 - \frac{2wl}{12EI}x^3 + \frac{w}{24EI}x^4$

the general soln

$y = CF + P.I$

$= A + Bx + \frac{wl^2}{4EI}x^2 - \frac{2wl}{12EI}x^3$

$+ \frac{w}{24EI}x^4$

$\frac{dy}{dx} = B + \frac{2wl^2x}{4EI} - \frac{6wlx^2}{12EI} + \frac{4w}{24EI}x^3$

at $y = 0$ and $x = 0$

$y = A + Bx + \frac{wl^2}{4EI}x^2 - \frac{2wl}{12EI}x^3 + \frac{2w}{24EI}x^4$

$0 = A$

$A = 0$

at $\frac{dy}{dx} = 0$ and $x = 0$

$0 = B + \frac{\cancel{2}wl^2x}{\cancel{4}_2EI} - \frac{\cancel{6}wlx^2}{\cancel{12}_2EI} + \frac{\cancel{8}w}{\cancel{24}_3EI}x^3$

$0 = B + \frac{wl^2}{2EI} - \frac{wlx^2}{2EI} + \frac{w}{3EI}x^3$

$B = 0$

General equation will be

$y = \frac{wl^2}{4EI}x^2 - \frac{\cancel{2}wl}{\cancel{12}_6EI}x^3 + \frac{w}{24EI}x^4$

$y = \frac{wl^2}{4EI}x^2 - \frac{wl}{6EI}x^3 + \frac{w}{24EI}x^4$

at $x = c$

$y = \frac{wl^4}{4EI} - \frac{wl^4}{6EI} + \frac{wl^4}{24EI}$

$y = \frac{6wl^4 - 4wl^4 + wl^4}{24EI}$

$y = \frac{\cancel{3}wl^4}{\cancel{24}EI}$

$y = \frac{wl^4}{8EI}$