

ASSIGNMENT II

1(i)  $m^2 + 4m + 5 = 0$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)}$$

$$m = \frac{-4 \pm \sqrt{16 - 20}}{2}$$

$$m = \frac{-4 \pm \sqrt{-4}}{2}$$

$$m = \frac{-4 \pm j^2}{2}$$

$$m = -2 \pm j$$

C.F  $y = e^{-2x} (A \cos \theta + B \sin \theta)$

P.I.  $y = (C \cos \theta + D \sin \theta)$

$$\frac{dy}{d\theta} = -C \sin \theta + D \cos \theta$$

$$\frac{d^2y}{d\theta^2} = -C \cos \theta - D \sin \theta$$

$$-C \cos \theta - D \sin \theta + 4(-C \sin \theta + D \cos \theta) + 5(C \cos \theta + D \sin \theta) = 6 \sin \theta$$

$$-C \cos \theta - D \sin \theta - 4C \sin \theta + 4D \cos \theta + 5C \cos \theta + 5D \sin \theta = 6 \sin \theta$$

$$-C \cos \theta - D \sin \theta - 4C \sin \theta + 4D \cos \theta + 5C \cos \theta + 5D \sin \theta = 6 \sin \theta$$

$$\cos \theta (-C + 4D + 5C) + \sin \theta (-D - 4C + 5D) = 6 \sin \theta$$

$$\cos \theta (4C + 4D) + \sin \theta (4D - 4C) = 6 \sin \theta$$

Comparing both sides

$$4C + 4D = 0 \quad \text{--- (1)}$$

$$-4C + 4D = 6 \quad \text{--- (2)}$$

$$8D = 6$$

$$D = 3/4$$

Substitute into (1)

$$4C + 3 = 0$$

$$C = -3/4$$

P.I.  $y = -\frac{3}{4} \cos \theta + \frac{3}{4} \sin \theta$   
 G.S.  $y = e^{-2\theta} (A \cos \theta + B \sin \theta) - \frac{3}{4} \cos \theta + \frac{3}{4} \sin \theta$

iii] At steady state  $\theta = \infty$ ,  $\frac{dy}{d\theta} = 0$

$$\frac{dy}{d\theta} = -2e^{-2\theta} (A \cos \theta + B \sin \theta) + e^{-2\theta} (-A \sin \theta + B \cos \theta) + \frac{3}{4} \sin \theta + \frac{3}{4} \cos \theta$$

$$0 = 0 + 0 + \frac{3}{4} \sin \theta + \frac{3}{4} \cos \theta$$

$$-\frac{3}{4} \sin \theta = \frac{3}{4} \cos \theta$$

$$-\sin \theta = \cos \theta$$

Divide through by  $\cos \theta$

$$\frac{-\sin \theta}{\cos \theta} = 1$$

$$-\tan \theta = 1$$

$$\tan \theta = -1$$

$$\theta = \tan^{-1}(-1)$$

$$\theta = -45^\circ$$

2] F.I.  $\frac{d^2y}{dx^2} = \frac{w}{2} (1-x)^2$

C.F

Auxiliary equation

$$m^2 = 0$$

$$m = 0$$

$m = 0$  twice

$$y = e^0 (A + Bx)$$

$$y = A + Bx$$

Assume P.I.

$$y = Cx^2 + Dx^3 + Fx^4$$

$$\frac{dy}{dx} = 2Cx + 3Dx^2 + 4Fx^3$$

$$\frac{dy}{dx^2} = c + 6Dx + 12Fx^2$$

$$EI(2c + 6Dx + 12Fx^2) = w/2(1-x)^2$$

$$2CEI - 6DEI x + 12FEI x^2 = w/2(1-x)^2$$

$$2CEI - 6DEI x + 12FEI x^2 = w/2(1^2 - 2Lx + x^2)$$

$$4CEI + 12DEI x + 24FEI x^2 = w(L^2 - 2Lx + x^2)$$

$$4CEI + 12DEI x + 24FEI x^2 = wL^2 - 2wLx + wx^2$$

Comparing Coefficient

$$x^2: 24FEI = w$$

$$F = \frac{w}{24EI}$$

$$x = 12DEI = -2wL$$

$$D = -\frac{wL}{6EI}$$

$$\text{Constant: } 4CEI = wL^2$$

$$C = \frac{wL^2}{4EI}$$

$$\therefore y = \frac{wL^2}{4EI} x^2 - \frac{wL}{6EI} x^3 + \frac{w}{24EI} x^4$$

$$\therefore \text{G.S} = y = A + Bx + \frac{wL^2}{4EI} x^2 - \frac{wL}{6EI} x^3 + \frac{w}{24EI} x^4$$

$$\frac{dy}{dx} = B + \frac{wL^2}{2EI} x - \frac{wL}{2EI} x^2 + \frac{w}{6EI} x^3$$

$$\text{at } \frac{dy}{dx} = 0, x=0$$

$$0 = B$$

$$\text{at } y=0, x=0$$

$$A = 0$$

G.S

$$y = \frac{wL^2}{4EI} x^2 - \frac{wL}{6EI} x^3 + \frac{w}{24EI} x^4$$

$$\text{at } x=L$$

$$y = \frac{wL^4}{4EI} - \frac{wL^4}{6EI} + \frac{wL^4}{24EI}$$

$$y = \frac{6wL^4 - 4wL^4 + wL^4}{24EI}$$

$$y = \frac{3wL^4}{24EI}$$

$$y = \frac{wL^4}{8EI}$$