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ENG381 ASSIGNMENT

1. $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

$m^2 - m - 2 = 0$

$(m-2)(m+1) = 0$

$\therefore m_1 = 2$ or $m_2 = -1$

$y = Ae^{2x} + Be^{-x}$

$PI = y = C$

$\frac{dy}{dx} = 0$

$\frac{d^2y}{dx^2} = 0$

$0^2 - 0 - 2C = 8$

$\frac{-2c}{-2} = \frac{8}{-2}$

$\therefore c = -4$

$\therefore y = Ae^{2x} + Be^{-x} - 4$

2. $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$m^2 - 4 = 0$

$m^2 = 4$

$m = \pm\sqrt{2}$

$\therefore m_1 = +2$ or $m_2 = -2$

$y = Ae^{2x} + Be^{-2x}$

$PI = y = Ce^{3x}$

$\frac{dy}{dx} = 3Ce^{3x}$

$\frac{d^2y}{dx^2} = 9Ce^{3x}$

$9Ce^{3x} + 0(3Ce^{3x}) - 4(Ce^{3x}) = 10e^{3x}$

$9C - 4C = 10$; $5C = 10$

$\therefore C = 2$ $\quad y = Ae^{2x} + Be^{-2x} + 2e^{3x}$

3 $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

$m^2 + 2m + 1 = 0$

$(m+1)(m+1) = 0$

$\therefore m_1 = -1$ and $m_2 = -1$

$y = e^{-x}(A + Bx)$

$PI = y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$4C - 4C + C = 1$

$C = 1$

$y = e^{-x}(A + B) + e^{-2x}$

4 $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

$m^2 + 25 = 0$

$m = \pm\sqrt{25}$

$m_1 = +5 \quad m_2 = -5$

$y = Ae^{5x} + Be^{-5x}$

$PI = y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2Cx + D$

$\frac{d^2y}{dx^2} = 2C$

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$25Cx^2 + 25Dx + (25E + 2C) = 5x^2 + x$

$25Cx^2 = 5x^2 \quad \therefore C = 1/5$

$25Dx = x \quad \therefore D = 1/25$

$25E + 2C = 0 \; ; \; 25E + 2(1/5) = 0 \; ; \; 25E = -2/5 \quad E = -2/125$

$y = Ae^{5x} + Be^{-5x} +$ ~~$\left(5x + \frac{x}{25} - \frac{6}{5}\right)$~~ $\left(\frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}\right)$

$y = Ae^{5x} + Be^{-5x} +$ ~~$\frac{1}{25}(125x^2 + x - 20)$~~ $\frac{1}{5}\left(x^2 + \frac{x}{5} - \frac{2}{25}\right)$

5 $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

$m^2 - 2m + 1 = 0$

$(m-1)(m-1) = 0$

$m_1 = 1$ and $m_2 = 1$

$CF = y = e^{x}(A + Bx)$

$PI = y = C\cos x + D\sin x$

$\frac{dy}{dx} = -C\sin x + D\cos x$

$\frac{d^2y}{dx^2} = -C\cos x - D\sin x$

$-C\cos x + D\sin x - 2(-C\sin x + D\cos x) + C\cos x + D\sin x) = 4\sin x$

~~$-C\cos x$~~ $+ D\sin x + 2C\sin x - 2D\cos x +$ ~~$C\cos x$~~ $+ D\sin x = 4\sin x$

$2D\sin x + 2C\sin x - 2D\cos x = 4\sin x$

$\sin x(2D + 2C) - 2D\cos x = 4\sin x$

$-2D\cos x = 0$

$D = 0$

$2D + 2C = 4$

$2(0) + 2C = 4 \quad \therefore C = 2$

$PI \Rightarrow y = 2\cos x$

$G.S \Rightarrow y = e^{x}(A + Bx) + 2\cos x$

6. $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

$3m^2 - 2m - 1 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{2 \pm \sqrt{(-2)^2 - 4(3)(-1)}}{2(3)}$

$= \frac{2 \pm \sqrt{4 + 12}}{6}$

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$$m = \frac{2 \pm \sqrt{16}}{6}$$

$$= \frac{2 \pm 4}{6}$$

$$= \frac{2+4}{6} \quad \text{and} \quad \frac{2-4}{6}$$

$m_1 = 1$ and $m_2 = -\frac{1}{3}$

CF $\Rightarrow y = Ae^{x} + Be^{x/3}$

P.I $= y = Cx^2 + Dx + E$

$$\frac{dy}{dx} = 2Cx + D$$

$$\frac{d^2y}{dx^2} = 2C$$

$3(2C) - 2(2Cx + D) - (Cx^2 + Dx + E) = 2x - 3$

$6C - 4Cx - 2D - Cx^2 - Dx - E = 2x - 3$

$Cx^2 - 4Cx - Dx + 6C - 2D - E = 2x - 3$

$Cx^2 = 0$

$\therefore C = 0$

$-4Cx - Dx = 2x$

$-4(0) - D = 2$

$\therefore D = -2$

$6C - 2D - E = -3$

$6(0) - 2(-2) - E = -3$

$0 + 4 - E = -3$

$\therefore E = 7$

P.I $\Rightarrow y = -2x + 7$

G.S $\Rightarrow y = Ae^{x} + Be^{x/3} - 2x + 7$

8. $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

$m^2 - 6m + 8 = 0$

$(m-4)(m-2) = 0$

$\therefore m_1 = 4$ and $m_2 = 2$

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$CF \Rightarrow y = Ae^{4x} + Be^{2x}$

$PI = y = Ce^{4x}$

$\frac{dy}{dx} = 4Ce^{4x}$

$\frac{d^2y}{dx^2} = 16Ce^{4x}$

$16Ce^{4x} - 6(4Ce^{4x}) + 8(Ce^{4x}) = 8e^{4x}$

$16Ce^{4x} - 24Ce^{4x} + 8Ce^{4x} = 8e^{4x}$

$16C - 24C + 8C = 8$

$\frac{0C}{0} = \frac{8}{0} \quad \therefore C = 0$

$\therefore G.S \Rightarrow y = Ae^{4x} + Be^{2x}$

6. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$; $x = 0, y = 1, \frac{dy}{dx} = -2$

$m^2 + 4m + 5 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)}$

$= \frac{-4 \pm \sqrt{16 - 20}}{2}$

$= \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm 2j}{2}$

$m = -2 \pm j$

$C.F \Rightarrow y = e^{-2x}(A\cos x + B\sin x)$

$PI \Rightarrow y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4ce^{-2x} + 4(-2ce^{-2x}) + 5(ce^{-2x}) = 2e^{-2x}$

$4c - 8c + 5c = 2$

$\therefore c = 2$

$G.S \Rightarrow y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

when $x = 0$ and $y = 1$

$1 = e^{-0}(A\cos 0 + B\sin 0) + 2e^{-2(0)}$

$1 = 1(A(1) + B(0)) + 2$

$1 = A + 2$

$\therefore A = -1$

$y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

Let $u = e^{-2x}$

$\frac{du}{dx} = -2e^{-2x}$

Let $v = A\cos x + B\sin x$

$\frac{dv}{dx} = -A\sin x + B\cos x$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$= e^{-2x}(-A\sin x + B\cos x) + (A\cos x + B\sin x) \cdot -2e^{-2x} - 4e^{-2x}$

When $\frac{dy}{dx} = -2$ and $x = 0$

$-2 = e^{-0}(+1\sin 0 + B\cos 0) + (-1\cos 0 + B\sin 0) \cdot -2e^{-0} - 4e^{-2(0)}$

$-2 = 1(0 + B) + (-1 + 0) \cdot -2e^{-0} - 4e^{0}$

$-2 = B + 2 - 4$

$\therefore B = -4 + 2 + 2 \quad \therefore B = 0$

~~$\therefore GS \Rightarrow y = e^{-2x}(-\cos x - 4\sin x) + 2$~~

$\therefore GS \Rightarrow y = e^{-2x}(-\cos x) + 2e^{-2x}$

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