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15/ENG02/007
CIVIL ENG.
ENG 381 (Maths)

Assignment 2

1.) $\frac{d^2y}{d\theta^2} + \frac{4dy}{d\theta} + 5y = 6\sin\theta$

convert equation to homogenous equation

$\frac{d^2y}{d\theta^2} + \frac{4dy}{d\theta} + 5y = 0$

$m^2 + 4m + 5 = 0$

$a = 1, \ b = 4, \ c = 5$

$m = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)}$

$= \frac{-4 \pm \sqrt{-4}}{2}$

$= \frac{-4 \pm j\sqrt{4}}{2}$

$= \frac{-4 \pm j2}{2}$

$m = -2 \pm j$

∴ complementary function,

$y = e^{-2\theta}(A\cos\theta + B\sin\theta)$

~~$y = C\cos\theta$~~

$\Rightarrow y = C\cos\theta + D\sin\theta$

$\frac{dy}{d\theta} = -C\sin\theta + D\cos\theta$

$\frac{d^2y}{d\theta^2} = -C\cos\theta - D\sin\theta$

Sub y, $dy/d\theta$ & $d^2y/d\theta^2$ into the eqn

$-C\cos\theta - D\sin\theta + 4[-C\sin\theta + D\cos\theta] + 5[C\cos\theta + D\sin\theta] = 6\sin\theta$

$-C\cos\theta - D\sin\theta - 4C\sin\theta + 4D\cos\theta + 5C\cos\theta + 5D\sin\theta = 6\sin\theta$

$-C + 4D + 5C = 0$

$4D + 4C = 0 \quad -(i)$

$-D + 4C + 5D = 6$

$4D + 4C = 6$ — (2)

equating both eqns:

$4D + 4C = 0$ — (1)

$-4D + 4C = 6$ — (2)

$8C = 6$

$C = 6/8$

$C = 3/4$

sub C into eqn (1)

$4D + 4(3/4) = 6$

$4D + 3 = 6$

$4D = 6 - 3$

$4D = 3$

$D = 3/4$,

∴ Particular Integral,

$y = 3/4\cos\theta + 3/4\sin\theta$

$y = e^{-2\theta}(A\cos\theta + B\sin\theta) + 3/4\cos\theta + 3/4\sin\theta$

~~$y =$~~

G.S = C.F + P.I

$y = e^{-2\theta}(A\cos\theta + B\sin\theta) + 3/4(\cos\theta + \sin\theta)$

at steady state,

$\theta = \alpha$, $dy/d\theta = 0$

$y = e^{-2\theta}(A\cos\theta + B\sin\theta) + 3/4(\cos\theta + \sin\theta)$

$\frac{dy}{d\theta} = -2e^{-2\theta}(-A\sin\theta + B\cos\theta) + 3/4(-\sin\theta + \cos\theta)$

$0 = -2e^{-2\alpha}(B\cos\alpha - A\sin\alpha) + 3/4(\cos\alpha - \sin\alpha)$

since, $e^{-\alpha} = 0$

$0 = 3/4\cos\alpha - 3/4\sin\alpha$

$\frac{3/4\sin\alpha}{3/4\cos\alpha} = \frac{3/4\cos\alpha}{3/4\cos\alpha}$

$\tan\alpha = 1$

$\alpha = \tan^{-1} = 1$

$\alpha = 45^\circ$

2.) $EI \frac{d^2y}{dx^2} = \frac{w}{2}(L-x)^2$

convert equation to homogeneous equation

$EI \frac{d^2y}{dx^2} = 0$

$EI\, m^2 = 0$

$m^2 = 0$

$m = \pm\sqrt{0}$

$m_1 = m_2 = 0$

$y = e^{\alpha x}(A + Bx)$

complementary function,

$y = A + Bx$

$y = Rx^2 + Sx^3 + Tx^4$

$\frac{dy}{dx} = 2Rx + 3Sx^2 + 4Tx^3$

$\frac{d^2y}{dx^2} = 2R + 6Sx + 12Tx^2$

$EI[2R + 6Sx + 12Tx^2] = \frac{w}{2}(L-x)^2$

$2REI + 6SxEI + 12Tx^2EI = \frac{w}{2}[L^2 - 2Lx + x^2]$

multiply by equation 2

$4REI + 12SxEI + 24Tx^2EI = WL^2 - 2WLx + Wx^2$

$24TEI = W$

$T = \frac{W}{24EI}$

$12SEI = -2WL$

$S = \frac{-2WL}{24EI}$

$y = \left[\frac{WL^2}{4EI}\right]x^2 - \left[\frac{WL}{6EI}\right]x^3 + \left[\frac{W}{24EI}\right]x^4$

$y = \frac{WL^2x^2}{4EI} - \frac{WLx^3}{6EI} + \frac{Wx^4}{24EI}$

$$y = \frac{6wl^2x^2 - 4wlx^3 + wx^4}{24EI}$$

$$y = \frac{M}{24EI}\left[6l^2x^2 - 4lx^3 + x^4\right] \quad \text{(Particular Integral)}$$

G.S = C.F + P.I

$$y = A + Bx + \frac{M}{24EI}\left[6l^2x^2 - 4lx^3 + x^4\right]$$

at $x = 0$, $y = 0$, $dy/dx = 0$

$$0 = A + B(0) + \frac{M}{24EI}\left[12l^2x - 12lx^2 + 4x^3\right]$$

$$0 = B + \frac{M}{24EI}\left[12l^2(0) - 12l(0)^2 + 4(0)^3\right]$$

$B = 0$

when $A = B = 0$

$$y = 0 + 0x + \frac{M}{24EI}\left[6l^2x^2 - 4lx^3 + x^4\right]$$

$$y = \frac{M}{24EI}\left[6l^2x^2 - 4x^3 + x^4\right]$$

when $x = l$

$$y = \frac{M}{24EI}\left[6l^4 - 4l^4 + l^4\right]$$

$$y = \frac{M}{24EI}\left[3l^4\right]$$

$$y = \frac{ML^4}{8EI}$$

$x = 0$, $dy/dx = 0$

$$\frac{dy}{dx} = A + B + \frac{M}{24EI}\left[12l^2x - 12lx^2 + 4x^3\right]$$

$$\frac{dy}{dx} = A + B + \frac{M}{24EI}\left[12l^2(0) - 12l(0)^2 + 4(0)^3\right]$$

$$\frac{dy}{dx} = A + B + \frac{M}{24EI}$$