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15/ENG04/056 ELECT/ELECT

① $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

$m^2 - m - 2 = 0$

$(m-2)m+2=0$

$m_1 = 2 \quad m_2 = -1$

CF $y = Ae^{2x} + Be^{-x}$

PI $y = c$

$\frac{dy}{dx} = 0$

$\frac{d^2y}{dx^2} = 0$

$\therefore 0 - 0 - 2c = 8$

$c = \frac{8}{-2}$

$c = -4$

$y = -4$

G.S $y = PI + CF$

$y = Ae^{2x} + Be^{-x} - 4$

(2) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$m^2 - 4 = 0$

$m^2 = 4$

$m = \pm 2$

CF : $y = A\cosh 2x + B\sinh 2x$

PI $\Rightarrow y = ce^{3x}$

$\frac{dy}{dx} = 3ce^{3x}$

$d^2y = 9ce^{3x}$

$9ce^{3x} - 4ce^{3x} = 10e^{3x}$

$5ce^{3x} = 10e^{3x}$

$5c = 10$

$c = 2$

$y = 2e^{3x}$

GS : $A\cosh 2x + B\sinh 2x + 2e^{3x}$

(3) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-3x}$

$m^2 + 2m + 1 = 0$

$(m+1)(m+1) = 0$

$m_1 = -1 \quad m_2 = -1$

CF $y = e^{-x}(A + Bx)$

PI $y = ce^{-2x}$

$= -2ce^{-2x}$

$\frac{dy}{dx} = 4ce^{-2x}$

$\frac{d^2y}{dx^2}$

$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$

$C = 1$

$\therefore y = e^{-2x}$

$y = CF + PI$

$y = e^{-2}(A + Bx) + e^{-2x}$

$\frac{d^2y}{dx^2} + 25y = 5x^2 + 2$

$m^2 + 25 = 0$

$m = -25$

$m = \pm 5j$

$y = A\cos 5x + B\sin 5x$

$PI = y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2Cx + D$

$\frac{d^2y}{dx} = 2C$

$2C + 25(Cx^2 + Dx + E) \; 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$25x^2 + 25Dx + 2C + 25E = 5x^2 + 2$

$25C = 5 \quad \text{——} ①$

$25D = 1$ — (2)

$2C + 25E = 0$ — (3)

$C = \frac{3}{25} = \frac{1}{5}$ $\quad D = \frac{1}{25}$

$E = -\frac{2}{25} = -\frac{2}{125}$

PI $y = \frac{x^2}{5} + \frac{x}{25} - \frac{2}{125}$

$\therefore$ GS $y = A\cos 5x + B\sin 5x + \frac{x^2}{5} + \frac{x}{25} - \frac{5}{125}$

(5) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 7\sin x$

$m^2 - 2m + 1 = 0$

$(m-1)(m-1) = 0$

$m_1 = 1 \quad m_2 = 1$

CF $y = e^x(A + Bx)$

$\Rightarrow -C\cos x - D\sin x - 3(-C\sin x + D\cos x) + C\cos x + D\sin x$

$= 4\sin x$

$\Rightarrow -C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x = 4\sin x$

$\Rightarrow -C\cos x + C\cos x + 2C\sin x - D\sin x + D\sin x - 2D\cos x = 4\sin x$

$\Rightarrow 2C\sin x - 2D\cos x = 4\sin x$

$\therefore C\sin x - D\cos x = 2\sin x$

$C = 2$

$-D = 0$

$\therefore D = 0$

$y = 2\cos x$

GS $y = e^{2x}(A + Bx) + 2\cos 2x$

(6) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$

$m^2 + 4m + 5 = 0$

$m = \frac{-4 \pm \sqrt{4^2 - 4 \times 5}}{2 \times 1}$

$= \frac{-4 \pm \sqrt{-4}}{2}$

$m = -2 \pm j$

$y = e^{-2x}(A\cos x + B\sin x)$

PI $y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$\rightarrow 4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

$Ce^{-2x} = 2e^{-2x}$

$\therefore C = 2$

PI $y = 2e^{-2x}$

GE $y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

given at $x = 0$ $y = 1$ $\frac{dy}{dx} = -2$

$1 = e^{-2(0)}(A\cos 0 + B\sin 0) + 2e^{2(0)}$

$A + 2 = 1 \quad \therefore A = -1$

$dy/dx = e^{-2x}(-A\sin x + B\cos x) + (A\cos x + B\sin x)$

$-2e^{-2x} - 4e^{-2x}$

$-1 = e^{-2(0)}(B) + (A) - 2 - 4$

$-2 = B - 2A - 4$

$-2 = B + 1 - 4$

$B = 1$

$\therefore y = e^{-2x}(-\cos x + \sin x) + 2e^{-4x}$

(3) $3\frac{d^2y}{dx} + 2\frac{dy}{dx} - y = 2x - 3$

$3m^2 - 2m - 1 = 0$

$m = \frac{-2 \pm \sqrt{4 - 4 \times 3 \times -1}}{6}$

$= \frac{-2 \pm \sqrt{16}}{6}$

$m = \frac{-2 \pm 4}{6}$

$m = -2/6 \pm 4/6 = -1/3 \pm 2/3$

$m_1 = \frac{-1}{3} + \frac{2}{3}$ $\quad m_2 = -1/2 - 2/6$

$m_1 = 1/3$ $\quad m_2 = -1$

CF $y = Ae^{x/3} + Be^{-x}$

$$dy/dx = e^{-2x}(-A\sin x + B\cos x) + (A\cos x + B\sin x)$$
$$- 2e^{-2x} - 4e^{-2y}$$

$$-1 = e^{-2(0)}(B) + (A) - 2 - 4$$

$$-2 = B - 2A - 4$$

$$-2 = B + 1 - 4$$

$$B = 1$$

$$\therefore y = e^{-2x}(-\cos x + \sin x) + 2e^{-2x}$$

(7) $3\frac{d^2y}{dx} + 2\frac{dy}{dx} - y = 2x - 3$

$$3m^2 - 2m - 1 = 0$$

$$m = \frac{-n \pm \sqrt{4 - 4 \times 3 \times -1}}{6}$$

$$= \frac{-2 \pm \sqrt{16}}{6}$$

$$m = \frac{-2 \pm 4}{6}$$

$$m = -2/6 \pm 4/6 = -1/3 \pm 2/3$$

$$m_1 = \frac{-1}{3} + \frac{2}{3} \qquad m_2 = -1/2 - 2/2$$

$$m_1 = 1/3 \qquad m_2 = -1$$

$$CF \quad y = Ae^{x/3} + Be^{-x}$$

P.I $y = C(x+D)$

$\frac{dy}{dx} = C$

$\frac{d^2y}{dx^2} = 0$

$5(a) - 2C(x+D) = 2x - 3$

$-2C - Cx + D = 2x - 3$

$-C(x) - 2C + D = 2x - 3$

$-C = 2$

$C = -2$

$-2C + D = -3$

$D = -3 + 2C$

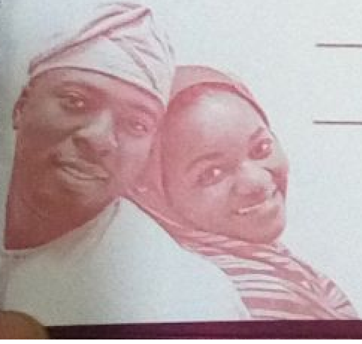
$D = -3 + 2(-2)$

$D = -3 - 4$

$D = -7$

$\therefore$ PI $= y = -2x - 7$

$y = Ae^{x/3} + Be^{-x} - 2x - 7$



8 $d^2y/dx^2 - 6\,dy/dx + 8y = 8e^{8x}$

$m^2 - 6m + 8 = 0$

$(m-4)(m-2) = 0$

$m_1 = 4 \quad m_2 = 2$

CF $y = Ae^{4x} + Be^{2x}$

$y = ce^{ax}$

$\rightarrow dy/dx = c(x \cdot 4e^{4x} \cdot e^{4x} \cdot 1)$

$= c(4xe^{4x} + e^{4x})$

$= 4cxe^{4x} + ce^{4x}$

$\rightarrow d^2y/dx = 4c(x \cdot 4e^{4x} + e^{4x} \cdot 1) + 4ce^{4x}$

$= 4c(4xe^{4x} + e^{4x}) + 4ce^{4x}$

$= 16cxe^{4x} + 4ce^{4x} + 4ce^{4x}$

$= 16cxe^{4x} + 8ce^{4x}$

$\Rightarrow 16cxe^{4x} + 8ce^{4x} - [6(cxe^{4x} + ce^{4x}] + 8(cxe^{4x}) = 8e^{4x}$

$= 16cxe^{4x} + 8ce^{4x} - 24cxe^{4x} - 6ce^{4x} + 8ce^{4x} = 8e^{4x}$

$\rightarrow 16cx + 8c - 24cx - 6c + 8cx = 8$

$2c = 8$

$c = 8/2$

$\therefore c = 4$

PI - $y = 4xe^{4x}$

GS $\therefore$ $y = Ae^{4x} + Be^{2x} + 4xe^{4x}$