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15/ENG06/032
MECHANICAL ENGINEERING
# ASSIGNMENT 1
ENG 381

1. $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

$m^2 - m - 2$

$(m^2 - 2m) + (m - 2) = 0$

$m(m-2) + 1(m-2) = 0$

$(m+1)(m-2) = 0$

$m = -1 \; ; \; m = 2$

~~y = Ae~~ ∴ The complementary function becomes

$y = Ae^{m_1 x} + Be^{m_2 x}$

$y = Ae^{-x} + Be^{2x}$

We assume the general form of PI to be

$y = C \quad \text{- - - - - (i)}$

$dy/dx = 0 \quad \text{- - - - (ii)}$

$d^2y/dx^2 = 0 \quad \text{- - - - (iii)}$

Recall the original eqn is $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

Subst eqn. i, ii, iii in the eqn to get

$0 - 0 - 2(C) = 8$

$-2C = 8$

$C = -4$

Recall complete eqn is

CF + PI

General soln is

$$y = Ae^{-x} + Be^{2x} - \frac{4}{2}.$$

2. $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$m^2 - 4 = 0$

$m^2 = 4$

$m = \pm 2$

CF:- $y = A\cosh 2x + B\sinh 2x$

Assumed PI.

$y = Ce^{3x} \quad \ldots \ldots (i)$

$dy/dx = 3Ce^{3x} \quad \ldots \ldots (ii)$

$d^2y/dx^2 = 9Ce^{3x} \quad \ldots \ldots (iii)$

Subst. (i), (ii), (iii) in general eqn. $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

$5Ce^{3x} = 10e^{3x}$

$C = \frac{10e^{3x}}{5e^{3x}}$

$C = 2$

$y = 2e^{3x}$

Complete eqn is CF + PI

General soln is: $y = A\cosh 2x + B\sinh 2x + 2e^{3x}$.

3. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

$m^2 + 2m + 1$

$(m^2 + m) + (m + 1) = 0$

$m(m+1) + 1(m+1) = 0$

$(m+1)(m+1) = 0$

$m = -1$ (twice)

C.F.: $y = e^{-x}(A + Bx)$

Assumed form of PI is

$y = Ce^{-2x}$ - - - (i)

$dy/dx = -2Ce^{-2x}$ - - - (ii)

$d^2y/dx^2 = 4Ce^{-2x}$ - - - (iii)

Subst. eqn. (i)(ii)(iii) into the general eqn.

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$\cancel{4Ce^{-2x}} - \cancel{4Ce^{-2x}} + Ce^{-2x} = e^{-2x}$

$Ce^{-2x} = e^{-2x}$

$C = 1$

$y = e^{-2x}$

Complete eqn is: CF + PI

General eqn therefore becomes $= e^{-x}(A + Bx) + e^{-2x}$

4. $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

$m^2 + 25 = 0$

$m^2 = -25$

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$m = \sqrt{-25}$

$m = \sqrt{-1} \times \sqrt{25}$

$m = j5$

$\therefore m = \pm j5.$

C.F. $y = A\cos 5x + B\sin 5x$

Assumed P.I.:

$y = Cx^2 + Dx + E \quad \text{------} \quad (i)$

$dy/dx = 2Cx + D \quad \text{----} \quad (ii)$

$d^2y/dx^2 = 2C \quad \text{----} \quad (iii)$

Subst eqn (i) (ii) & (iii) in $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

Equating coefficients we have;

$[x^2]$ $25C = 5$

$C = 1/5 \quad \text{---} \quad (i)$

$[x]$ $25D = 1$

$D = 1/25 \quad \text{---} \quad (ii)$; subst eqn (i) & (ii) in $2C + 2E$

$2C + 25E = 0$

$2(1/5) + 25E = 0$

$25E = -2/5$

$E = -2/5 \times 1/25$

$E = -2/125$

$P.I. = \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

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~~PI~~ Multiply through by 125

$25x^2 + 5x - 2$

$\frac{1}{125}(25x^2 + 5x - 2)$

General eqn becomes (CF + PI)

$y = A\cos 5x + B\sin 5x + \frac{1}{25}(25x^2 + 5x - 2)$

5. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

CF: $m^2 - 2m + 1 = 0$

$(m^2 - m) - (m + 1) = 0$

$m(m-1) - 1(m-1) = 0$

$(m-1)(m-1) = 0$

$m = 1$ (twice)

CF $\therefore$ $y = e^x[A + Bx]$

Assumed PI $= C\cos x + D\sin x$

$y = C\cos x + D\sin x$ ---- (i)

$\frac{dy}{dx} = -C\sin x + D\cos x$ ---- (ii)

$\frac{d^2y}{dx^2} = -C\cos x - D\sin x$ ---- (iii)

Subst eqn. (i), (ii) & (iii) in $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

~~$-C\sin x + D\cos x - 2($~~

$-C\cos x - D\sin x - 2(-C\sin x + D\cos x) + C\cos x + D\sin x = 4\sin x$

$-C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x = 4\sin x$

$-C\cos x - 2D\cos x + C\cos x - D\sin x + 2C\sin x + D\sin x = 4\sin x$

$\cos x(-C - 2D + C) + \sin x(-D + 2C + D) = 4\sin x$

$\cos x(-2D) + \sin x(2C) = 4\sin x$

Comparing coefficients:

$-2D = 0$

$D = 0$

$2C = 4$

$\therefore C = 2$

$\therefore P.I = 2\cos x + 0(\sin x)$

$y = 2\cos x$

The general soln then becomes

$y = e^{x}(A + Bx) + 2\cos x$

6. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$

given that @ $x = 0$, $y = 1$ & $dy/dx = -2$

$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$

$m^2 + 4m + 5 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)}$

$m = \frac{-4 \pm \sqrt{-4}}{2}$

$m = \frac{-4}{2} \pm j \frac{2}{2}$

$m = -2 \pm j$

$y = e^{-2x}(A\cos x + B\sin x)$

$\therefore$ P.I, $y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

Subst. $dy/dx$ & $d^2y/dx^2$ in general eqn get.

$4Ce^{-2x} + 4(-2Ce^{-2x}) + 5(Ce^{-2x}) = 2e^{-2x}$

$4Ce^{-2x} + 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

$Ce^{-2x}(4 - 8 + 5) = 2e^{-2x}$

$C\cancel{e^{-2x}} = 2\cancel{e^{-2x}}$

$C = 2$

$P.I = y = 2e^{-2x}$

General soln then becomes

$y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

at $x = 0$ $y = 1$

$\therefore$ $1 = e^{-2(0)}(A\cos(0) + B\cos(0)) + 2e^{-2(0)}$

$1 = 1(A + 0) + 2$

$1 = A + 2$

$A = -1$

$\therefore$ at $x = 0$ $dy/dx = -2$

$\frac{dy}{dx} = -2e^{-2x}(-A\sin x + B\cos x) + 2e^{-2x}$

Subst $dy/dx = -2$ get.

2015/2

$-2 = -2e^{-2(0)}(-A\sin(0) + B\cos(0)) - 2e^{-2(0)}$

$-2 = -2(0 + B) - 2$

$-2 = -2B - 2$

$-2B = 0$

$B = 0$

$y = e^{-2x}(-\cos x + 0\sin x) + 2e^{-2x}$

$y = -e^{-2x}\cos x + 2e^{-2x}$

$\therefore$ general eqn beomes

$y = e^{-2x}(-\cos x + 2)$

7. $3\frac{d^2y}{dx^2} - \frac{2dy}{dx} - y = 2x - 3$

$\frac{3d^2y}{dx^2} - \frac{2dy}{dx} - y = 0$

$3m^2 - 2m - 1 = 0$

$(3m^2 - 3m) + (m - 1) = 0$

$3m(m-1) + 1(m-1) = 0$

$(3m+1)(m-1) = 0$

$3m + 1 = 0$ ; $m - 1 = 0$

$m = -1/3$ $\quad$ $m = 1$

C.F; $y = Ae^{-1/3x} + Be^{x}$

P.I, $y = Cx + D$

$dy/dx = C$

$d^2y/dx^2 = 0$

$3(0) + -2(C) - (Cx + D) = 2x - 3$

$0 - 2C - Cx - D = 2x - 3$

$-2C - D - Cx = 2x - 3$

Comparing coefficients

for $x$

$-Cx = 2x$

$C = -2$

$-2C - D = -3$

$-2(-2) - D = -3$

$4 - D = -3$

$-D = -3 - 4$

$D = 7.$

$\therefore$ P.I : $y = -2x + 7$

The general soln. then becomes

G.S =

$y = Ae^{-1/3x} + Be^{x} - 2x + 7.$

8. $\frac{d^2y}{dx^2} - \frac{6dy}{dx} + 8y = 8e^{4x}$

$\frac{d^2y}{dx^2} - \frac{6dy}{dx} + 8y = 0$

$m^2 - 6m + 8 = 0$

$(m^2 - 2m) - (4m + 8) = 0$

$m(m-2) - 4(m-2) = 0$

$(m-4)(m-2) = 0$

$m_1 = 4 \ ; \ m_2 = 2$

2015/20

$\therefore \; y = Ae^{4x} + Be^{2x}$

P.I: $y = Ce^{4x}$

$\frac{dy}{dx} = 4Ce^{4x}$

$\frac{d^2y}{dx^2} = 16Ce^{4x}$

$16Ce^{4x} - 6(4Ce^{4x}) + 8(Ce^{4x}) = 8e^{4x}$

$Ce^{4x}(16 - 24 + 8) = 8e^{4x}$

$Ce^{4x}(0) = 8e^{4x}$

$C = \frac{8e^{4x}}{e^{4x}(0)}$

$y = 0.$

The general soln becomes

$y = Ae^{4x} + Be^{2x}.$