AMOD, EMMANUEL
15/ENG06/011
ENG 381

1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8.$

Soln.

Equate the L.H.S to 0.

$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$

Auxiliary equation becomes

$m^2 - m - 2 = 0$

$m^2 - 2m + m - 2 = 0$

$m(m-2) + 1(m-2) = 0$

$(m+1)(m-2) = 0$

$m = -1$ or 2.

Real & different root.

$y = Ae^{-x} + Be^{2x}.$

Assumed P.I.

$f(x) = 8.$

$y = C \quad \ldots (1)$

$\frac{dy}{dx} = 0 \quad \ldots (2)$

$\frac{d^2y}{dx^2} = 0 \quad \ldots (3)$

Substitute (1) (2) & (3) into the original equation.

$0 - 0 - 2(C) = 8$

$-2C = 8$

$C = -4.$

The general solution becomes

$y = Ae^{-x} + Be^{2x} - 4.$

2. $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

To find CF

$m^2 - 4 = 0$

$m^2 = 4$

$m = \pm\sqrt{4}$

$m = \pm 2.$

$y = Ae^{2x} + Be^{-2x}$

Assumed PI.

$f(x) = 10e^{3x}$

$y = Ce^{3x} \ldots (1)$

$\frac{dy}{dx} = 3Ce^{3x} \ldots (2).$

$\frac{d^2y}{dx^2} = 9Ce^{3x} \ldots (3)$

put (1) (2) & (3) into the real eqn.

$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

$5Ce^{3x} = 10e^{3x}.$

Divide both sides by e^{3x}.

$5C = 10$

$C = \frac{10}{5}$

$C = 2.$

Assumed PI.

$y = 2e^{3x}$

The general solution = CF + PI.

$y = Ae^{2x} + Be^{-2x} + 2e^{3x}$

3. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

To find CF.

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0.$

Auxiliary equation second

$m^2 + 2m + 1 = 0.$

$m = -1 \ \& \ -1.$

Real & equal root.

$y = e^{-x}(A + Bx).$

Assumed PI.

$f(x) = e^{-2x}$

$y = Ce^{-2x} \ldots (1).$

$\frac{dy}{dx} = -2Ce^{-2x} \ldots (2).$

$\frac{d^2y}{dx^2} = 4Ce^{-2x} \ldots (3).$

put (1) (2) & (3) into the original equation

$4Ce^{-2x} + 2[-2Ce^{-2x}] + Ce^{-2x} = e^{-2x}$

$\cancel{4Ce^{-2x}} - \cancel{4Ce^{-2x}} + Ce^{-2x} = e^{-2x}$

$Ce^{-2x} = e^{-2x}.$

divide both sides by e^{-2x}.

$C = 1$

PI = $y = e^{-2x}.$

The general solution (CF + PI)

$\Rightarrow y = e^{-x}(A + Bx) + e^{-2x}.$

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4. $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

Soln.

To get CF.

$\frac{d^2y}{dx^2} + 25y = 0$.

Auxiliary equation becomes:

$m^2 + 25 = 0$.

$m^2 = -25$

$m = \pm\sqrt{25}$

$m = \pm\sqrt{-1} \cdot \sqrt{25}$

$m = \pm 5j$

but $m = \alpha \pm j\beta$.

$y = A\cos 5x + B\sin 5x$.

Assumed PI.

$f\,x = 5x^2 + x$.

$y = Cx^2 + Dx + E \cdots (1)$

$\frac{dy}{dx} = 2Cx + D \cdots (2)$

$\frac{d^2y}{dx^2} = 2C \cdots (3)$

put (1), (2) (3) into real equation.

$2C - 2(Cx^2 + Dx + E) = 5x^2 + x$.

$2C - 2Cx^2 - 2Dx - 2E = 5x^2 + x$

comparing coefficients.

$x^2: -2C = 5$.

$C = \frac{5}{-2}$

$x: -2D = 1$

$D = \frac{1}{-2}$

Constant: $2C - 2E = 0$.

$2\left(\frac{-5}{2}\right) - 2(E) = 0$.

$-5 - 2E = 0$

$-5 = 2E$.

$E = \frac{-5}{2}$

$y = -\frac{5}{2}x^2 - \frac{x}{2} - \frac{5}{2}$.

The general equation becomes (CF+PI).

$\Rightarrow$

$A\cos 5x + B\sin 5x - \frac{5x^2}{2} - \frac{x}{2} - \frac{5}{2}$ //

5. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x.$

Soln.

To find CF.

$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0.$

Auxiliary equation becomes

$m^2 - 2m + 1 = 0.$

$m = 1 \; \& \; 1.$

$y = e^x(A + Bx).$

Assumed PI.

$f(x) = 4\sin x.$

$y = C\cos x + D\sin x \; \cdots \; (1).$

$\frac{dy}{dx} = -C\sin x + D\cos x \; \cdots \; (2)$

$\frac{d^2y}{dx^2} = -C\cos x - D\sin x \; \cdots \; (3).$

put (1), (2) & (3) into the original eqn.

$-C\cos x - D\sin x - 2[-C\sin x + D\cos x]$

$+ C\cos x + D\sin x = 4\sin x.$

$-\cancel{C\cos x} - \cancel{D\sin x} + 2C\sin x - 2D\cos x$

$+ \cancel{C\cos x} + \cancel{D\sin x} = 4\sin x$

Comparing coefficients.

$\sin x: -2C = 4 \;;\; C = -2.$

$\cos x: -2D = 0 \;;\; D = 0.$

$y = -2\cos x + 0\sin x. \;;\; y = -2\cos x$

General solution becomes (CF + PI).

$y = e^x[A + Bx) - 2\cos x$ //

6. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-3x}.$

Soln.

To find CF.

$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0.$

$m^2 + 4m + 5 = 0.$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{-4 \pm \sqrt{4^2 - 4\times1\times5}}{2\times1}$

$\frac{-4 \pm \sqrt{16 - 20}}{2} = \frac{-4 \pm \sqrt{-4}}{2}$

$= \frac{-4}{2} \pm \frac{\sqrt{-4}}{2}$

$= -2 \pm \sqrt{-1} \times \frac{\sqrt{4}}{2}.$

$= -2 \pm j \cdot \frac{2}{2}.$

$= -2 \pm j \cdot 1.$

$= -2 \pm j.$

$m = \alpha \pm j\beta.$

$\therefore \alpha = -2.$

$\beta = 1.$

$y = e^{-2x}[A\cos x + B\sin x].$

Assumed PI.

$f(x) = 2e^{-3x}$

$y = Ce^{-3x} \; \cdots \; (1).$

$\frac{dy}{dx} = -3Ce^{-3x} \; \cdots \; (2).$

$\frac{d^2y}{dx^2} = 9Ce^{-3x} \; \cdots \; (3).$

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put (1) (2) & (3) into original eqn.

$9ce^{-3x} + 4(-3ce^{-3x}) + 5(ce^{-3x}) = 2e^{-3x}$

$9ce^{-3x} - 12ce^{-3x} + 5ce^{-3x} = 2e^{-3x}$

$2c\cancel{e^{-3x}} = 2\cancel{e^{-3x}}$

$c = 1.$

$y = e^{-3x}$

General solution becomes CF + PI.

$y = e^{-2x}[A\cos x + B\sin x] + e^{-3x}.$

8.

7. $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3.$

soln.

To find CF.

$3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 0.$

$3m^2 - 2m - 1 = 0.$

Using quadratic formula.

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-2) \pm \sqrt{-2^2 + 4 \times 3 \times 1}}{2 \times 3}$

$= \frac{2 \pm \sqrt{4 + 12}}{6}$

$= \frac{2 \pm \sqrt{16}}{6}$

$= \frac{2 \pm 4}{6}$

$= \frac{2+4}{6}$ or $\frac{2-4}{6}$

$= \frac{6}{6} = 1$ or $-\frac{2}{6} = \frac{-1}{3}.$

$m = 1$ or $\frac{-1}{3}.$

$y = Ae^{x} + Be^{-1/3x}.$

Assumed PI.

$f(x) = 2x - 3.$

$y = cx + D \quad \ldots (1).$

$\frac{dy}{dx} = c \quad \ldots (2).$

$d^2y/dx^2 = 0 \quad \ldots (3).$

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put ① ② ③ into original equation.

$3(0) - 2(c) - (cx + D) = 2x - 3$

$0 - 2c - (x - D) = 2x - 3$

By comparing coefficients:

$x: -c = 2$

$k: -2c - D = -3$

$c = -2$

$-2(-2) - D = -3$

$4 - D = -3$

$4 + 3 = D$

$7 = D$

$y = -2x + 7$

general equation becomes:

CF + PI

$y = Ae^{x} + Be^{-1/3x} + (-2x + 7)$

8. $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

Soln.

$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0$

Auxiliary equation becomes:

$m^2 - 6m + 8 = 0$

$m = 4$ or $m = 2$

$y = Ae^{4x} + Be^{2x}$

Assumed PI.

$f(x) = 8e^{4x}$; $y = Ce^{4x}$

since e^{4x} is in the CF, multiply ~~divide~~ PI by x.

$y = xCe^{4x}$ ... ①

$\frac{dy}{dx} = x4Ce^{4x} + Ce^{4x}$ ... ②

$\frac{d^2y}{dx^2} = x16Ce^{4} + 4Ce^{4} \cdot 1 + Ce^{4x} + 4Ce^{4x}$

$= 16xCe^{4x} + 4Ce^{4x}$ ... ③

put ① ② & ③ into real eqn.

$[16xCe^{4x} + 4Ce^{4x}] - 6(4xCe^{4x} + Ce^{4x}) + 8(xCe^{4x}) = 8e^{4x}$

$16xCe^{4x} + 4Ce^{4x} - 24xCe^{4x} - 6Ce^{4x} + 8xCe^{4x} = 8e^{4x}$

$4Ce^{4x} - 6Ce^{4x} = 8e^{4x}$

$-2Ce^{4x} = 8e^{4x}$

$C = 8/2 \quad C = -4$

$y = 4e^{4x}$

General solution becomes CF + PI.

$y = Ae^{4x} + Be^{2x} + (-4e^{4x})$