AMUDO, EMMANUEL
15/ENG06/011
ENG 381 (2).

1. $\frac{d^2y}{dx^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta$.

<u>soln</u>

$m^2 + 4m + 5 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

$m = \frac{-4 \pm \sqrt{4^2 - (4\times1\times5)}}{2\times1}$

$m = \frac{-4 \pm \sqrt{-4}}{2}$

$m = \frac{-4 \pm \sqrt{4}\cdot\sqrt{-1}}{2}$

$m = -\frac{4}{2} \pm \frac{2i}{2}$

$m = -2+i$ or $m = -2-i$.

$CF = c_1 e^{(-2+i)\theta} + c_2 e^{(-2-i)\theta}$

$y_n = c_1 e^{-2\theta + i\theta} + c_2 e^{-2\theta - \theta i}$

$y_n = c_1 e^{-2\theta} \cdot e^{i\theta} + c_2 e^{-2\theta} \cdot e^{-\theta i}$

$y_n = e^{-2\theta}[c_1 e^{i\theta} + c_2 e^{-\theta i}]$.

$y_n = e^{-2\theta}[A\cos\theta + B\sin\theta]$.

$y_p = A\cos\theta + B\sin\theta$.

$y'_p = -A\sin\theta + B\cos\theta$.

$y''_p = -A\cos\theta - B\sin\theta$.

$-A\cos\theta - B\sin\theta + 4[-A\sin\theta + B\cos\theta] + 5A\cos\theta + 5B\sin\theta = 6\sin\theta$.

$-A\cos\theta - B\sin\theta - 4A\sin\theta + 4B\cos\theta + 5A\cos\theta + 5B\sin\theta = 6\sin\theta$.

$4A\cos\theta + 4B\sin\theta - 4A\sin\theta + 4B\cos\theta = 6\sin\theta$.

$(-4A + 4B)(\sin\theta) + 4A + 4B(\cos\theta) = 6\sin\theta$.

$-4A + 4B = 6$

$4A + 4B = 0$

$8B = 6$

$B = 6/8$

$B = 3/4$.

but $4A = -4B$

$A = -B$

$A = -3/4$

$y_p = -3/4 \cos\theta + 3/4 \sin\theta.$

$y = y_n + y_p$

$y = e^{-2\theta}[A\cos\theta + B\sin\theta] + 3/4\sin\theta - 3/4\cos\theta.$

Steady state equation.

$y'p = 0$

$yp' = 3/4\cos\theta + 3/4\sin\theta = 0.$

$\Rightarrow 3/4\cos\theta + 3/4\sin\theta = 0.$

$3/4\cos\theta = -3/4\sin\theta$

$\cos\theta = -\sin\theta$

$\frac{\sin\theta}{\cos\theta} = \frac{-\cos\theta}{\cos\theta}.$

$\tan\theta = -1.$

$\theta = \tan^{-1}(-)$

$\theta = -45^\circ$

2.

$EI\, d^2y/dx^2 = \omega/2\,(l-x)^2.$

$EI \cdot m^2 = 0.$

$m^2 = 0.$

$m = \pm\sqrt{0}$

$m = \pm 0$

$y = e^{0x}(A + Bx)$ CF.

$y = A + Bx.$

To get PI.

$y = Ex^2 + Fx^3 + Gx^4$

$y' = 2Ex^2 + 3Fx^2 + 4Gx^3$

$y'' = 2E + 6Fx + 12Gx.$

$EI[2E + 6Fx + 12Gx^2] = \omega/2\,(l-x^2).$

$2E\cdot EI + 6EIFx + 12EIGx^2 = \frac{\omega}{2}(l^2 - 2lx + x^2).$

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$4EIE + 12EIFx + 24EIGx^2 = w(l^2 - 2lx + x^2)$.

$24GEI = w$

$G = \frac{w}{24EI}$ ... (1)

$12EIF = -2wl$

$F = \frac{-2wl}{12EI}$

$= \frac{-wl}{6EI}$ ... (2).

$4EIE = wl^2$

$E = \frac{wl^2}{4EI}$.

$$y = \left[\frac{wl^2}{4EI}\right]x^2 - \left[\frac{wl}{6EI}\right]x^3 + \left[\frac{w}{24EI}\right]x^4.$$

$$= \frac{wl^2x^3}{4EI} - \frac{wlx^3}{6EI} + \frac{wx^4}{24EI}.$$

$$= \frac{6wl^2x^2 - 4wlx^2 + wx^4}{24EI}.$$

$$= \frac{w}{24EI}\left[6l^2x^2 - 4lx^3 + x^4\right)$$

$$y = A + Bx + \frac{w}{24EI}\left[6l^2x^2 - 4lx^3 + x^4\right].$$ ... general eqn.

at $y = 0$; $x = 0$ $dy/dx = 0$.

$$0 = A + B(0) + \frac{w}{24EI}\left[6l^2(0) - 4l(0) + 0\right].$$

$$\frac{dy}{dx} = B + \frac{w}{24EI}\left[6l^2(0) - 4l(0) + 0\right].$$

$$0 = B + \frac{w}{24EI}\left[12(0) - 12(0) + 4(0)\right].$$

$B = 0$.

Particular solution.

$$y = \frac{w}{24EI}\left[6l^2x^2 - 4lx^3 + x^4\right].$$

$$y = \frac{wx^2}{24EI}\left[6l^2 - 4lx + x^2\right]$$

$$y = \frac{wx^2}{24EI}\left[x^2 - 4lx + 6l^2\right].$$

when $x = l$

$$y = \frac{wl^3}{24EI}\left[l^2 - 4l^2 + 6l^2\right].$$

$$y = \frac{wl^4}{8EI}$$