NELSON-UDU 'A' ELAH
16/ENG07/016
Petroleum Engineering

ENG 281

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Department of Chemical & Petroleum

1(a) $\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4)\ (\sin(\cos x))}{x - \pi/2} \right]$

(b) $\lim_{x \to \pi/2} \ln \left(\exp \frac{(3x^2 + 2x - 1)}{x + 1} \right)$

(c) $\lim \cos \left[\sin^{-1} \frac{(x - 2)}{x - \sqrt{3}} \right]$

(d) $\lim_{x \to 4} \left(\frac{x^2 - 8x + 16}{x^2 - 5x + 4} \right)$

Solution
for the numerator

(a) let $u = x^2 - \pi/4$

$V = \sin(\cos x)$

$dy/dx = u \cdot dv/dx + v\, du/dx$

for $u = x^2 - \pi/4$

$du/dx = 2x$

for $V = \sin(\cos x)$

let $w = \cos x$, $dw/dx = -\sin x$

$v = \sin w$, $dv/dw = \cos w$

$dv/dx = dw/dx \times dv/dw = -\sin x \cos w$

$dv/dx = -\sin \cos(\cos x))$

$dy/dx = \left[x^2 - \pi/4\right] - \cos(\cos x)\sin x + \sin(\cos x)$

$dy/dx = \left[x^2 - \pi/4\right] - \cos(\cos x)\sin x + \sin(\cos x) \cdot 2x$

for the Denominator

$b = x - \pi/2$

$db/dx = 1$

②

$$\therefore \frac{(x^2 - \pi/4)(-\cos(\cos x)\sin x) + \sin(\cos x)\, 2x}{1}$$

$$\lim_{x \to \pi/2} = \left[\left(\frac{\pi^2}{2} - \frac{\pi}{4}\right)\left(-\cos(\cos \tfrac{\pi}{2})\sin\tfrac{\pi}{2} + \sin(\cos\tfrac{\pi}{2})\, \tfrac{2\pi}{2}\right)\right.$$

$$= \left(\pi^2/4 - \pi/4\right)(-1) + 0$$

$$= \left(\frac{\pi^2 - \pi}{4}\right) - 1$$

$$= \frac{-\pi^2 + \pi}{4}$$

$$= \frac{\pi(-\pi + 1)}{4}$$

(b) $\cancel{\ln}\left[\cancel{\exp}\ \frac{(3x^2 + 2x - 1)}{x+1}\right]$

$$= \lim_{x \to \pi/2} \left[\frac{3(\pi/2)^2 + 2(\pi/2) - 1}{(\pi/2) + 1}\right]$$

$$= \frac{\frac{3\pi^2}{4} + \frac{\pi}{1} - \frac{1}{1}}{\frac{\pi + 2}{2}}$$

$$= \frac{3\pi^2 + 4\pi + 4}{\cancel{4}\,2} \times \frac{\cancel{2}}{\pi + 2}$$

$$= \frac{3\pi^2 + 4\pi - 4}{2(\pi + 2)}$$

$$= \frac{(\pi/1 - 2/3)(\pi + 2)}{2(\pi + 2)}$$

$$= \frac{3\pi - 2}{3} \times \cancel{\tfrac{1}{2}}$$

$$= \frac{3\pi - 2}{6}$$

c $\lim_{x \to 2+\sqrt{3}} \cos \left[\sin^{-1} \frac{(2+\sqrt{3}) - 2}{(x - \sqrt{3})} \right]$

$$= \cos \left[\sin^{-1} \left(\frac{\sqrt{3}}{2} \right) \right]$$

$$= \cos 60$$

$$= \frac{1}{2}$$

(d) $\lim_{x \to 4} \left[\frac{2x - 8}{2x - 5} \right]$

$$= \lim_{x \to 4} \left[\frac{2(4) - 8}{2(4) - 5} \right]$$

$$= \frac{8 - 8}{8 - 5}$$

$$= \frac{0}{3}$$

$$= 0$$

(2) Determine whether each of the following series is convergent.

(a) $\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6} + \cdots$

soln

$$\frac{2}{2^p} + \frac{2}{3^p} + \frac{2}{4^p} + \frac{2}{5^2} + \cdots$$

when $p = 2$ we get

$$\frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \frac{2}{5^2} + \cdots$$

Since

$$\frac{2}{2\times3} < \frac{2}{2^2}, \quad \frac{2}{3\times4} < \frac{2}{3^2}, \quad \frac{2}{4\times5} < \frac{2}{4^2}$$

$\therefore$ the series is convergent.

④

$$\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \cdots$$

soln

for comparison test using standard series, when $P=2$

$$\frac{2}{1^p} + \frac{2}{2^p} + \frac{2}{3^2} + \frac{2}{4^2} + \cdots$$

Since $P > 1$ the series converges.

$\therefore$ the given series is convergent.

$$u_n = \frac{1+2n^2}{1+n^2}$$

soln

$$u_n = \frac{1+2n^2}{1+n^2}$$

so $\lim_{n \to \infty} u_n = \lim_{n \to \infty} \left[\frac{1/n^2 + 2n^2/n^2}{1/n^2 + n^2/n^2}\right]$

$$= \left[\frac{1/n^2 + 2/1}{1/n^2 + 1/1}\right]$$

$$= \frac{0+2}{0+1}$$

$$= 2,$$

since $u_n \neq 0$

$\lim_{n \to \infty}$

$\therefore$ the series is Divergent.

(3) Find the range of values of x for which the series below is absolutely convergent

$$\frac{x}{27} + \frac{x^2}{125} + \cdots + \frac{x^n}{(2n+1)^3}$$

Soln

$$|u_n| = \frac{x^n}{(2n+1)^3}$$

$$\frac{|u_{n+1}|}{|u_n|} = \frac{x^{n+1}}{(2n+3)^3} \times \frac{(2n+1)^3}{x^n}$$

$$= \frac{\cancel{x^n} \cdot x}{(2n+3)^3} \times \frac{(2n+1)^3}{\cancel{x^n}}$$

$$= \frac{x(2n+1)^3}{(2n+3)^3}$$

$$= \frac{x(2n+1)(2n+1)(2n+1)}{(2n+3)(2n+3)(2n+3)}$$

$$= \frac{x(8n^3+12n^2+6n+1)}{8n^3+36n^3+54n+27}$$

$$= \frac{x\left[\frac{8n^3}{n^3}+\frac{12n^2}{n^3}+\frac{6n}{n^3}+\frac{1}{n^3}\right]}{\left(\frac{8n^3}{n^3}+\frac{36n^2}{n^3}+\frac{54n}{n^3}+\frac{27}{n^3}\right)}$$

$$= x\left[\frac{8+\frac{12}{n}+\frac{6}{n^2}+\frac{1}{n^3}}{8+\frac{36}{n}+\frac{54}{n^2}+\frac{27}{n^3}}\right]$$

$$= x\left[\frac{8+0+0+0}{8+0+0+0}\right]$$

$$= \frac{8x}{8}$$

$$= x;$$

$$\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = x$$

for absolute convergent $\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| < 1$

$\therefore$ series convergent when $-1 \le x \le 1$

(6)

(4) Evaluate using L'Hopital's Rule

$$\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right]$$

Soln.

$$= \lim_{x \to 0} \left[\frac{\sin x - \cos x}{x} \right]$$

$$= \lim_{x \to 0} \left[\frac{\cos x + \sin x}{3x^2} \right]$$

$$= \lim_{x \to 0} \left[\frac{-\sin x + \cos x}{6x} \right]$$

$$= \lim_{x \to 0} \left[\frac{-\cos x - (-\sin x)}{6} \right]$$

$$= \lim_{x \to 0} \left[\frac{-\cos x - \sin x}{6} \right]$$

$$= \frac{-1 + 0}{6}$$

$$= -1/6$$