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MECHATRONICS
16/ENG05/025
ENG 281 ASSIGNMENT

The parametric equations of a curve are as given in equations (1) and (2)

$$x = \cos t + t\sin t$$
$$y = \sin t - t\cos t$$

In terms of t determine

i) An expression for the radius of curvature (R) and an expression for the coordinates (h, k) of the centre of curvature

Solution

$$x = \cos t + t\sin t$$
$$y = \sin t - t\cos t$$

$$\frac{dx}{dt} = \cancel{-\sin t} + \cancel{\sin t} + t\cos t$$

$$\frac{dx}{dt} = t\cos t$$

$$\frac{dy}{dt} = \cos t - \cos t + t\sin t$$

$$\frac{dy}{dt} = t\sin t$$

$$\frac{dy}{dx} = \frac{dy}{dt} - \frac{dx}{dt}$$

$$\frac{dy}{dx} = \frac{\cancel{t}\sin t}{\cancel{t}\cos t} = \frac{\sin t}{\cos t}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right)\left(\frac{dt}{dx}\right)$$

Applying Quotient rule

$u = \sin t \qquad v = \cos t$

$$\frac{V\,du/dx - U\frac{dv}{dx}}{V^2}$$

$$\frac{du}{dt} = \cos t$$

$$\frac{dx}{dt} = -\sin t$$

$$\frac{d^2y}{dx} = \frac{\cos t \cdot \cos t - (\sin t)(-\sin t)}{\cos^2 t} \times \frac{dt}{dx}$$

$$= \frac{\cos^2 t + \sin^2 t}{\cos^2 t} \times \frac{dt}{dx}$$

From trigonometric identies

$$\sin^2\theta + \cos^2\theta = 1$$

$$\frac{d^2y}{dx^2} = \frac{1}{\cos^2 t} \times \frac{1}{t\cos t}$$

$$\frac{d^2y}{dx} = \frac{1}{t\cos^3 t}$$

$$R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2}$$

$$R = \left[1 + \left(\frac{\sin t}{\cos t}\right)^2\right]^{3/2} \div \frac{1}{t\cos^3 t}$$

$$R = \left[1 + \frac{\sin^2 t}{\cos^2 t}\right]^{3/2} \times t\cos^3 t$$

$$R = \left[\frac{\cos^2 t + \sin^2 t}{\cos^2 t}\right]^{3/2} \times t\cos^3 t$$

$$R = \frac{1}{(\cos^2 t)^{3/2}} \times t\cos^3 t$$

$$R = \frac{1}{\cancel{\cos^3 t}} \times t\,\cancel{\cos^3 t}$$

$$R = t$$

The expression of Radius of curvature (R) is

Recall

$h = x_1 - R\sin\theta$

$k = y_1 + R\cos\theta$

$R = t \quad , \theta = t$

$x_1 = \cos t + t\sin t$

$y_1 = \sin t - t\cos t$

$h = \cos t + t\sin t - (t\sin t)$

$h = \cos t$

$k = \sin t - t\cos t + t\cos t$

$k = \sin t$

The expressions for the coordinates of (h, k) of the centre of curvature is $(\cos t, \sin t)$