

AD E DIPE E SECUM

15/ENC 06/001

MECHANICAL ENGINEERING

1.
$$\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta$$

We observe homogeneity

$$y'' + 4y' + 5y = 6\sin\theta$$

where $y = e^{k\theta}$

$$y'' = k^2 e^{k\theta}$$

$$y' = k e^{k\theta}$$

$$k^2 e^{k\theta} + 4(k e^{k\theta}) + 5(e^{k\theta}) = 6\sin\theta$$

$$k^2 e^{k\theta} + 4k e^{k\theta} + 5e^{k\theta} = 0$$

$$e^{k\theta} (k^2 + 4k + 5) = 0$$

$$k^2 + 4k + 5 = 0$$

$$k^2 + 4k = -5$$

$$k^2 + 4k + \left(\frac{1 \times 4}{2}\right)^2 = -5 + \left(\frac{1}{2} \times 4\right)^2$$

$$k^2 + 4k + 4 = -5 + 4$$

$$(k+2)(k+2) = -1$$

$$k+2 = \sqrt{-1}$$

$$k_1 = -2+i \text{ and } k_2 = -2-i$$

$$y_n = C_1 y_1 + C_2 y_2$$

$$y_1 = e^{k_1 \theta} = e^{(-2+i)\theta} = e^{-2\theta} \cdot e^{i\theta}$$

$$y_2 = e^{k_2 \theta} = e^{(-2-i)\theta} = e^{-2\theta} \cdot e^{-i\theta}$$

$$y_n = C_1 e^{-2\theta} \cdot e^{i\theta} + C_2 e^{-2\theta} \cdot e^{-i\theta}$$

$$y_n = e^{-2\theta} (C_1 \cos\theta + C_2 \sin\theta)$$

Assume $y_p = A \cos\theta + B \sin\theta$

$$y_p' = -A \sin\theta + B \cos\theta$$

$$y_p'' = -A \cos\theta - B \sin\theta$$

$$-A \cos\theta - B \sin\theta + 4(-A \sin\theta + B \cos\theta) + 5(A \cos\theta + B \sin\theta) =$$

$$6 \sin\theta$$

$$-A \cos \theta - B \sin \theta - 4A \sin \theta + 4B \cos \theta + 5A \cos \theta + 5B \sin \theta = 6 \sin \theta$$

$$(4A - 4B) \cos \theta + (4B - 4A) \sin \theta = 6 \sin \theta$$

$$(4A + 4B) \cos \theta + (4B - 4A) \sin \theta = 6 \sin \theta$$

$$4B - 4A = 6$$

$$4A + 4B = 0$$

$$4B - 4A = 6$$

$$4B + 4A = 0$$

$$\frac{8B}{8} = \frac{6}{8}$$

$$B = \frac{3}{4}$$

$$4\left(\frac{3}{4}\right) - 4A = 6$$

$$12/4 - 4A = 6$$

$$4A = 6 - 12/4$$

$$A = -3/4$$

$$y_p = -3/4 \cos \theta + 3/4 \sin \theta$$

$$y = y_n + y_p$$

$$y = e^{-2\theta} (C_1 \cos \theta + C_2 \sin \theta) - 3/4 \cos \theta + 3/4 \sin \theta$$

ii) at $\theta = 0$ and $dy/d\theta = 0$

$$\frac{dy}{d\theta} = e^{-2\theta} [-C_1 \sin \theta + C_2 \cos \theta] + [C_1 \cos \theta + C_2 \sin \theta] - 2e^{-2\theta}$$

$$+ 3/4 \sin \theta + 3/4 \cos \theta$$

at $\theta = 0$ and $dy/d\theta = 0$

$$0 = 3/4 \sin \theta + 3/4 \cos \theta$$

$$-\cos \theta = \sin \theta$$

$$\sin \theta = 1$$

$$-\cos \theta$$

$$-\tan \theta = 1$$

$$\theta = \tan^{-1}(-1)$$

$$\theta = -45^\circ$$

$$2. \quad EI \frac{d^2y}{dx^2} = \frac{w}{2}(L-x)^2$$

$$EI m^2 = 0$$

$$m^2 = 0$$

$$m = \pm 0$$

$$y = e^{-x}(A+Bx)$$

$$y = A+Bx$$

$$\text{Assume } y_p = Px^2 + Qx^3 + Rx^4$$

$$y_p' = 2Px + 3Qx^2 + 4Rx^3$$

$$y_p'' = 2P + 6Qx + 12Rx^2$$

$$EI[2P + 6Qx + 12Rx^2] = \frac{w}{2}[L-x]^2$$

$$2PEI + 6QEIx + 12REIx^2 = \frac{w}{2}[L^2 - 2Lx + x^2]$$

$$4PEI + 12QEIx + 24REIx^2 = w[L^2 - 2Lx + x^2]$$

$$24REL = w$$

$$R = \frac{w}{24EI}$$

$$24EI$$

$$12QEIx = -2wL$$

$$Q = \frac{-2wL}{12EI} = -\frac{wL}{6EI}$$

$$4PEI = wL^2$$

$$P = \frac{wL^2}{4EI}$$

$$y = \left[\frac{wL^2}{4EI} \right] x^2 - \left[\frac{wL}{6EI} \right] x^3 + \left[\frac{w}{24EI} \right] x^4$$

$$y = \frac{wL^2 x^2}{4EI} - \frac{wL x^3}{6EI} + \frac{w x^4}{24EI}$$

$$y = \frac{w x^2}{24EI} (3x^2 + 4Lx + 6L^2)$$