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MECHANICAL ENGINEERING
15/ENG06/001

1. $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

CF

$m^2 + m - 2m - 2 = 0$

$m(m+1) - 2(m+1)$

$m = -1$ or 2

$y = Ae^{-x} + Be^{2x}$

PI

Assumed PI : $y = c$

$\frac{dy}{dx} = 0$

$\frac{d^2y}{dx^2} = 0$

Substituting in the original equation.

$0 - 0 - 2c = 8$

$-2c = 8$

$c = -4$

$y = -4$

The general solution is.

CF + PI

$y = Ae^{-x} + Be^{2x} - 4$

2) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

CF

comparing with

$d^2y/dx^2 - n^2y = 0$

$n^2 = 4$

$n = 2$

$y = A\cosh 2x + B\cosh 2x$

assumed PI

$y = ce^{3x}$

assumed PI

$y = ce^{3x}$

$\frac{dy}{dx} = 3ce^{3x}$

~~assum~~ $\frac{d^2y}{dx^2} = 9ce^{3x}$

Substituting in the original equation.

$\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$9ce^{3x} - 4ce^{3x} = 10e^{3x}$

$5ce^{3x} = 10e^{3x}$

$c = \frac{10e^{3x}}{5e^{3x}}$

$c = 2$

$y = 2e^{3x}$

general solution

$y = A\cosh 2x + B\cosh 2x + 2e^{3x}$

3) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

CF: $m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$m(m+1) + 1(m+1)$

$m = -1$ twice.

$y = e^{-x}[A + Bx]$

PI

assumed PI: $y = ce^{-2x}$

$\frac{dy}{dx} = -2ce^{-2x}$

$\frac{d^2y}{dx^2} = 4ce^{-2x}$

Substituting in the original equation.

$4ce^{-2x} - 4ce^{-2x} + ce^{-2x} = e^{-2x}$

$ce^{-2x}[4 - 4 + 1] = e^{-2x}$

$ce^{-2x} = e^{-2x}$

$c = \frac{e^{-2x}}{e^{-2x}}$

$C = \frac{1}{\cancel{\ }}$

The PI is

$y = \frac{e^{-2x}}{\cancel{\ }}$

The general solution.

$y = e^{-x}[A + Bx] + e^{-2x}$

4) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

CF : comparing with

$\frac{d^2y}{dx^2} + n^2y = 0$

$n^2 = 25$

$n = 5$

$y = A\cos 5x + B\cos 5x$

PI : assumed PI : $y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2Cx + D$

$\frac{d^2y}{dx^2} = 2C$

Substitute in the original equation.

$2C + 25[Cx^2 + Dx + E] = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$25Cx^2 + 25Dx + 25E + 2C = 5x^2 + x$

$25C = 5 \quad [x^2]$

$C = 1/5$

$25D = 1 \quad [x]$

$D = 1/25$

$25E + 2C = 0$

$25E = -2/5$

$E = -2/5 \times 1/25$

$E = -2/50 = -1/25$

PI

$y = \frac{x^2}{5} + \frac{x}{25} - \frac{1}{25}$

The general solution.

$y = A\cos 5x + B\cos 5x + \frac{x^2}{5} + \frac{x}{25} - \frac{1}{25}$

5) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

CF.

$m^2 - 2m + 1 = 0$

$m^2 - m - m + 1 = 0$

$m[m-1] - 1[m-1] = 0$

$m = 1$ twice.

$y = e^x[A + Bx]$

PI : assumed PI

$y = C\cos x + D\sin x$

$\frac{dy}{dx} = -C\sin x + D\cos x$

$\frac{d^2y}{dx^2} = -C\cos x - D\sin x$

Substituting in the original equation

$-C\cos x - D\sin x - 2[-C\sin x + D\cos x] + C\cos x + D\sin x = 4\sin x$

$-C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x = 4\sin x$

$\cos x[-C - 2D + C] + \sin x[-D + 2C + D] = 4\sin x$

$2C = 4 \quad [\sin x]$

$C = 2$

$-2D = 0 \quad [\cos x]$

$D = 0$

PI

$y = 2\cos x$

The general solution is.

$y = e^x[A + Bx] + 2\cos x$

6) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$

CF

$m^2 + 4m + 5 = 0$

$\frac{-4 \pm \sqrt{16-20}}{2}$

$\frac{-4 \pm \sqrt{-4}}{2}$

$\frac{-4 \pm j\sqrt{4}}{2}$

$= -2 \pm j$

$y = e^{-2x}[A\cos x + B\sin x]$

PI

$y = ce^{-2x}$

$\frac{dy}{dx} = -2ce^{-2x}$

$\frac{d^2y}{dx^2} = 4ce^{-2x}$

Substituting in the original equation.

$4ce^{-2x} + 4(-2ce^{-2x}) + 5ce^{-2x} = 2e^{-2x}$

$4ce^{-2x} - 8ce^{-2x} + 5ce^{-2x} = 2e^{-2x}$

$ce^{-2x}[4 - 8 + 5] = 2e^{-2x}$

$ce^{-2x} = 2e^{-2x}$

$c = 2$

PI

$y = 2e^{-2x}$

general solution.

$y = e^{-2x}[A\cos x + B\sin x] + 2e^{-2x}$

$x = 0;\ y = 1$

$1 = [A] + 2$

$\therefore A = 1 - 2$

$A = -1$

$x = 0,\ \frac{dy}{dx} = -2$

$\frac{dy}{dx} = e^{-2x}[-A\sin x + B\cos x] - 2e^{-2x}[A\cos x + B\sin x] - 4e^{-2x}$

$-2 = [0 + B] - 2[A] - 4$

$-2 = B + 2 - 4$

$-2 = B - 2$

$B = 0$

$y = e^{-2x}(-\cos x) + 2e^{-2x}$

$y = -e^{-2x}\cos x + 2e^{-2x}$

$y = e^{-2x}(2 - \cos x)$

7. $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

CF

$3m^2 - 2m - 1 = 0$

$\frac{+2 \pm \sqrt{4+12}}{6}$

$\frac{+2 \pm \sqrt{16}}{6} = \frac{2 \pm 4}{6}$

$m = 1$ or $-\frac{1}{3}$

$y = Ae^{x} + Be^{-\frac{1}{3}x}$

PI

$y = Cx + B$

$\frac{dy}{dx} = C$

$\frac{d^2y}{dx^2} = 0$

substituting in the original equation.

$0 - 2C - Cx - B = 2x - 3$

$-Cx + (-2C - B) = 2x - 3$

$-C = 2 \quad [x]$

$C = -2$

$-2C - B = -3$

$4 - B = -3$

$B = 4 + 3 = 7$

$y = -2x + 7$

general solution.

$y = Ae^{x} + B^{-x/3} - 2x + 7$

8 $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

CF

$m^2 - 6m + 8 = 0$

$m^2 - 4m - 2m + 8 = 0$

$m(m-4) - 2(m-4)$

$m = 4$ or 2

$y = Ae^{4x} + Be^{2x}$

PI

$y = Cxe^{4x}$

$\frac{dy}{dx} = 4Cxe^{4x} + Ce^{4x}$

$\frac{d^2y}{dx^2} = 16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x}$

$16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} - 24Cxe^{4x} - 6Ce^{4x} + 8Cxe^{4x} = 8e^{4x}$

$Cxe^{4x}[16 - 24 + 8] + Ce^{4x}[4 + 4 - 6] = 8e^{4x}$

$\cancel{Cxe^{4x}[0]} + 2Ce^{4x} = 8e^{4x}$

$C = \frac{8e^{4x}}{2e^{4x}}$

$C = 4$

PI $:= 4$

$y = 4xe^{4x}$

$y = \underline{\underline{Ae^{4x} + Be^{2x} + 4xe^{4x}}}$