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Engineering mathematics

i) Evaluate

a) $$\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4)\sin(\cos x)}{x - \pi/2} \right]$$

solution

$$\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4)\sin(\cos x)}{x - \pi/2} \right]$$

substituting $\pi/2$ directly

$$\left[\frac{((\pi/2)^2 - \pi/4)\sin(\cos \pi/2)}{[\pi/2 - \pi/2]} \right] \to \text{Undefined}$$

Therefore Using l'hopital's rule

$$\lim_{x \to \pi/2} \left[\frac{f(x)}{g(x)} \right] = \lim_{x \to \pi/2} \left[\frac{f'(x)}{g'(x)} \right]$$

$\frac{dv}{dw} \times \frac{dx}{}$

Taking the numerator

$$y = [(x)^2 - \pi/4] \sin(\cos x)$$

$u = x^2 - \pi/4 \quad \therefore \quad du/dx = 2x$

$V = \sin(\cos x)$

Using the function of a function [let $w = \cos x \therefore V = \sin w$]

$$\frac{dv}{dx} = \frac{dv}{dw} \times \frac{dw}{dx}$$

where $w = \cos x$ $\quad \therefore \quad dw/dx = -\sin x$

$V = \sin w$ $\quad dv/dw = \cos w$

$$\frac{dv}{dx} = \cos w \times -\sin x$$

$\therefore \frac{dv}{dx} = -\sin x \cos(\cos x)$

Recall product rule

$y = uv$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$= (x^2 - \pi/4) - \sin x \cos(\cos x) + \sin[\cos(x)]2x$

$dy/dx = -(x^2 - \pi/4)\sin x \cos(\cos x) + 2x \sin(\cos(x))$

for denominator

$y = x - \pi/2 \qquad dy/dx = 1$

$\therefore \lim_{x \to \pi/2} \left[\frac{f'(x)}{g'(x)}\right] \therefore \lim_{x \to \pi/2} \left[-(x^2 - \pi/4)\sin \cos(\cos x) + 2x \sin[\cos(x)]\right]$

$= (\pi/2)^2 - \pi/4 \sin\cos[\cos \pi/2] + 2\,\pi/2 \sin[\cos \pi/2]$

Recall

$\pi = 180^\circ$

$\pi/2 = 90^\circ$

$= [\pi/2]^2 - \pi/4 \sin\cos(\cos 90) + 2[\pi/2)\sin[\cos 90)$

$= [\pi/2]^2 - \pi/4 \sin\cos(0) + 2(\pi/2)\sin 0$

$= [\pi/2]^2 - \pi/4 \sin 90 \cdot 1 + 2[\pi/2] \times 0$

$= [-[\pi/2]^2 - \pi/4 (1 \cdot 1) + 0]$

$\frac{dy}{dx} = -[\pi/2]^2 - \pi/4$

B $\lim_{x \to \pi/2} \ln \left[\exp \frac{(3x^2+2x-1)}{x+1} \right]$

solution

Recall $\ln \exp = \log_e = 1$

$$\lim_{x \to \pi/2} \ln\left[\exp \frac{(3x^2+2x-1)}{x+1}\right] = \lim_{x \to \pi/2} \left[\frac{3x^2+2x-1^2}{x+1}\right]$$

Using quadratic Equation

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$a = 3 \quad b = +2 \quad c = -1$

$$\frac{+2 \pm \sqrt{2^2-4(3-1)}}{2(3)}$$

$$= \frac{2+\sqrt{4+12}}{6} \; , \; \frac{2-\sqrt{4+12}}{6}$$

$$= \frac{2\sqrt{16}}{6} \; , \; \frac{2-\sqrt{16}}{6}$$

$$= \frac{2+4}{3} \; , \; \frac{2-4}{3}$$

$$= \frac{6}{6} \; , \; \frac{-2}{6}$$

$$= 1 \; , \; \frac{-1}{3}$$

$$\lim_{x \to \pi/2} \left[\frac{3x^2+2x-1}{x+1}\right] = \lim_{x \to \pi/2} \left[\frac{\cancel{(x+1)}(x-1/3)}{\cancel{(x+1)}}\right]$$

$$\lim_{x \to \pi/2} \left[x - 1/3\right]$$

$$= \frac{\pi}{2} - \frac{1}{3}$$

c) $\lim\limits_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\frac{(x-2)}{(x-\sqrt{3})}\right]$

Solution

$$\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\frac{(x-2)}{(x-\sqrt{3})}\right]$$

$$= \cos\left[\sin^{-1}\frac{2+\sqrt{3}-2}{2+\sqrt{3}-\sqrt{3}}\right]$$

$$= \cos\left[\sin^{-1}\frac{\sqrt{3}}{2}\right]$$

$$= \cos 60$$

$$= 0 \cdot 5$$

d $\lim\limits_{x \to 4}\left[\frac{x^2-8x+16}{x^2-5x+4}\right]$

solution

using L'hopitals rule

$\lim\limits_{x \to 4}\left[\frac{x^2-8x+16}{x^2-5x+4}\right]$ becoms $\lim\limits_{x \to 4}\left[\frac{2x-8}{2x-5}\right]$

$$\lim_{x \to 4}\left[\frac{2x-8}{2x-5}\right] = \frac{2(4)-8}{2(4)-5} = \frac{8-8}{8-5} = \frac{0}{3} = 0$$

2a ~~2~~ $\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \cdots$

$$U_n = \frac{2}{(n+1)(n+2)}$$

$$U_n = \frac{2}{n^2+2n+n+2}$$

$$U_n = \frac{2}{n^2+3n+2}$$

Using Comparison test

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \cdots \frac{1}{n^2}$$

Comparing the 1st, 2nd, 3rd, 4th and 5th term

$$\frac{2}{2\times3} < \frac{1}{1^2}$$

$$\frac{2}{3\times4} < \frac{1}{2^2}$$

$$\frac{2}{4\times5} < \frac{1}{3^2}$$

The series is convergent for the 1st, 2nd and 3rd term and 4th terms

23) $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + ----$

Solution

The nth term for this series $U_n = \frac{2}{n^2}$

Using DRT

$$\lim_{n \to \infty} \left[\frac{U_{n+1}}{U_n} \right]$$

where $U_{n+1} = \frac{2}{(n+1)^2}$

$$\lim_{n \to \infty} \left[\frac{\cancel{2}}{(n+1)^2} \times \frac{n^2}{\cancel{2}} \right]$$

$$\lim_{n \to \infty} \left[\frac{n^2}{(n+1)^2} \right]$$

$$\lim_{n \to \infty} \left[\frac{1}{1 + {}^2/n + {}^1/n^2} \right]$$

$$\left[\frac{1}{1+0+0} \right] = 1 \quad \text{(inconclusive)}$$

using Comparison test

Standard P series : $\frac{1}{n^p}$

$$\frac{2}{n^2} > \frac{1}{n^1} \quad \left[\text{for the 8th term } \frac{2}{64} > \frac{1}{8^p} \right]$$

~~then~~

Therefore the series is said to be divergent

2c $U_n = \frac{1+2n^2}{1+n^2}$

solution

$U_n = \frac{1+2n^2}{1+n^2}$

dividing through by highest power of n

$\lim_{n \to \infty} \left[U_n = \frac{1/n^2 + 2n^2/n^2}{1/n^2 + n^2/n^2} \right] \left[\lim = \frac{1/n^2 + 2}{1/n^2 + 1} \right]$

$\lim_{n \to \infty} \left[\frac{1/n^2 + 2}{1/n^2 + 1} \right] = \frac{0+2}{0+1} = \frac{2}{1} = 2$

∴ When $U_n \neq 0$ the series is divergent

The series is therefor divergent