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ENG381

$$1. \quad i \quad \frac{d^2 y}{d\theta^2} + 4 \frac{dy}{d\theta} + 5y = 6 \sin \theta$$

$$m^2 + 4m + 5 = 0$$

$$m = -2 \pm j$$

$$\alpha = -2 \quad \beta = 1$$

$$C.F. \Rightarrow y = e^{-2\theta} (A \cos \theta + B \sin \theta)$$

$$P.I. \Rightarrow y = C \cos \theta + D \sin \theta$$

$$\frac{dy}{d\theta} = -C \sin \theta + D \cos \theta$$

$$\frac{d^2 y}{d\theta^2} = -C \cos \theta - D \sin \theta$$

$$-C \cos \theta - D \sin \theta - 4C \sin \theta + 4D \cos \theta + 5C \cos \theta + 5D \sin \theta = 6 \sin \theta$$

$$\cos \theta (-C + 4D + 5C) + \sin \theta (-D - 4C + 5D) = 6 \sin \theta$$

$$4D + 4C = 0$$

$$4D - 4C = 6$$

$$C = -\frac{3}{4}, \quad D = \frac{3}{4}$$

$$P.I. \Rightarrow y = -\frac{3}{4} \cos \theta + \frac{3}{4} \sin \theta$$

$$G.S. \Rightarrow y = e^{-2\theta} (A \cos \theta + B \sin \theta) - \frac{3}{4} \cos \theta + \frac{3}{4} \sin \theta$$

$$ii \quad \frac{dy}{d\theta} = \frac{3}{4} \sin \theta + \frac{3}{4} \cos \theta$$

$$\frac{3}{4} \sin \theta + \frac{3}{4} \cos \theta = 0$$

$$\frac{3}{4} \sin \theta = -\frac{3}{4} \cos \theta$$

$$\frac{\sin \theta}{\cos \theta} = -\frac{\cos \theta}{\cos \theta}, \quad \tan \theta = -1$$

$$\theta = \tan^{-1}(-1) = -45^\circ$$

$$\theta \text{ at steady state} = -45^\circ$$

$$2. \quad EI \frac{d^2 y}{dx^2} = \frac{W}{2} (L-x)^2$$

$$EI m^2 = 0$$

$$m = 0$$

$$C.F. \Rightarrow y = e^{0x} (A + Bx)$$

$$y = A + Bx$$

$$P.I. \Rightarrow y = Cx^2 + Dx^3 + Ex^4$$

$$\frac{dy}{dx} = 2Cx + 3Dx^2 + 4Ex^3$$

$$\frac{d^2 y}{dx^2} = 2C + 6Dx + 12Ex^2$$

$$EI (2C + 6Dx + 12Ex^2) = \frac{W}{2} (L-x)^2$$

$$2EI(2C + 6Dx + 12Ex^2) = W(L-x)^2 = W(L^2 - 2Lx + x^2)$$

$$4EIC + 12DEIx + 24EI \cdot Ex^2 = WL^2 - 2WLx + Wx^2$$

$$24EI(E) = W$$

$$E = \frac{W}{24EI}$$

$$12DEI = -2WL$$

$$D = \frac{-2WL}{12EI}$$

$$4CEI = WL^2$$

$$C = \frac{WL^2}{4EI}$$

$$P.I. \Rightarrow y = \left( \frac{WL^2}{4EI} \right) x^2 + \left( \frac{-WL}{6EI} \right) x^3 + \left( \frac{W}{24EI} \right) x^4$$

$$y = \frac{WL^2 x^2}{4EI} - \frac{WL x^3}{6EI} + \frac{W x^4}{24EI}$$

$$= \frac{6WL^2 x^2 - 4WL x^3 + W x^4}{24EI}$$

$$= \frac{W (6L^2 x^2 - 4L x^3 + x^4)}{24EI}$$

$$= \frac{W (6L^2 x^2 - 4L x^3 + x^4)}{24EI}$$

$$G.S. \Rightarrow y = A + Bx + \frac{W (6L^2 x^2 - 4L x^3 + x^4)}{24EI}$$

$$24EI$$

$$\text{at } y=0; x=0; \frac{dy}{dx} = 0$$

$$0 = A + B(0) + \frac{w}{24EI} (5L^2(0)^2 + (-4(0)^3) + 0^4)$$

$$A = 0$$

$$\frac{dy}{dx} = \frac{B + w}{24EI} (12L^2x - 12Lx^2 + 4x^3)$$

$$0 = B + \frac{w}{24EI} (12L^2(0) - 12L(0)^2 + 4(0)^3)$$

$$B = 0$$

$$P.S \Rightarrow y = \frac{w}{24EI} (6L^2x^3 - 4Lx^3 + x^4)$$

$$x = L$$

$$y = \frac{w}{24EI} (6L^4 - 4L^4 + L^4)$$

$$= \frac{w}{24EI} (3L^4)$$

$$\therefore y = \frac{wL^4}{8EI}$$