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PETROLEUM ENGINEERING


Question 01

The model of a system is given in the equation below:

$$\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta.$$

i) Obtain an expression for $y$ as a function of $\theta$.

ii) Neglecting the complementary function of the general solution, with the aid of Ms Excel, plot the response of the system for $\theta = 0$ to $270°$.

iii) Using the expression obtained in (i), estimate the value of $\theta$ at a steady state.

Solution:

$$\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta$$

Let $y = e^{k\theta}$, $y' = ke^{k\theta}$, $y'' = k^2e^{k\theta}$

Assume homogeneity:

$$k^2e^{k\theta} + 4ke^{k\theta} + 5e^{k\theta} = 0$$

$$k^2 + 4k + 5 = 0$$

By completing the squares,

$$k^2 + 4k = -5$$

$$k^2 + 4k + 2^2 = -5 + 4$$

$$(k+2)^2 = -1$$

$$k + 2 = \pm\sqrt{-1}$$

$$\therefore k = \pm i - 2$$

$k_1 = i - 2$, $k_2 = -i - 2$

but $y = C_1e^{k_1\theta} + C_2e^{k_2\theta}$

$$= C_1e^{i\theta}\cdot e^{-2\theta} + C_2e^{-i\theta}\cdot e^{-2\theta}$$

$$y = e^{-2\theta}(C_1e^{i\theta} + C_2e^{-i\theta})$$

$$\therefore y = e^{-2\theta}(C_1\cos\theta + C_2\sin\theta)$$ — homogenous equation

Let $y_{nm} = A\sin\theta + B\cos\theta$

$y' = A\cos\theta - B\sin\theta$, $y'' = -A\sin\theta - B\cos\theta$

$\therefore -A\sin\theta - B\cos\theta + 4A\cos\theta - 4B\sin\theta + 5A\sin\theta + 5B\cos\theta = 6\sin\theta$

$\cancel{8A - 6}\ \sin\theta(-A - 4B + 5A) = 6\sin\theta$ [collecting like terms]

$\cancel{-B + 4A + 5B\}} = 0$ $\qquad 4A - 4B = 6 \longrightarrow (i)$

$4A + 4B = 0 \longrightarrow (ii)$

Solving simultaneously

$+4B + 4A = 6$

$+8A = 6$

$\therefore \cancel{B = -\frac{3}{4}}$ $\therefore A = \frac{3}{4}$

but $4A + 4B = 0$

$4\left(\frac{3}{4}\right) + 4B = 0$

$\therefore B = -3/4$.

~~Then, $A = -6$~~

~~Putting the value of A in equ. (i)~~

~~$-5B - 5B = 6$~~

~~$-10B = 6$~~

~~$B = -\frac{6}{10} = -\frac{3}{5}$~~

Hence, $y_{non} = \frac{3}{4}\sin\theta - \frac{3}{4}\cos\theta \longrightarrow$ non homogenous equation.

but $y = y_{homo} + y_{non}$

$y = e^{-2\theta}(C_1\cos\theta + C_2\sin\theta) + \frac{3}{4}\sin\theta - \frac{3}{4}\cos\theta$.

iii. At $\theta = \alpha$, $\frac{dy}{d\theta} = 0$

but $\frac{dy}{d\theta} = \frac{d}{d\theta}\left(C_1\cos\theta e^{-2\theta} + C_2\sin\theta e^{-2\theta} + \frac{3}{4}\sin\theta - \frac{3}{4}\cos\theta\right)$

$\frac{dy}{d\theta} = C_1\left(\cos\theta \cdot -2e^{-2\theta} + e^{-2\theta} \cdot -\sin\theta\right) + C_2\left(\sin\theta \cdot -2e^{-2\theta} + e^{-2\theta} \cdot \cos\theta\right) + \frac{3}{4}\cos\theta + \frac{3}{4}\sin$

$\frac{dy}{d\theta} = C_1\left(-2e^{-2\theta}\cos\theta - e^{-2\theta}\sin\theta\right) + C_2\left(-2e^{-2\theta}\sin\theta + e^{-2\theta}\cos\theta\right) + \frac{3}{4}\cos\theta +$

$\therefore \cancel{\frac{dy}{d\theta}}$ but at $\theta = \alpha$, $\frac{dy}{d\theta} = 0$

$0 = \frac{3}{4}\sin\theta + \frac{3}{4}\cos\theta$

$\frac{3}{4}\sin\theta = -\frac{3}{4}\cos\theta$

$\therefore \sin\theta = -\cos\theta$

$\therefore \cancel{\tan} -1 = \frac{\sin\theta}{\cos\theta} = \tan\theta$ [dividing through by $\cancel{\sin\theta}$ $\cos\theta$]

$\tan\theta = 1$

$\theta = \tan^{-1}(-1)$

$\therefore \theta = -45°$

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Question 02

The equation of bending for a horizontal cantilever having length L with load w per unit length is given in the equation below as:

$$EI\frac{d^2y}{dx^2} = \frac{w}{2}(L-x)^2,$$

where E, I, w and L are constants. If $y=0$ and $\frac{dy}{dx}=0$ at $x=0$, using the auxiliary equation method, find y as a function of x. Hence, evaluate the value of y when $x=L$.

Solution.

$EIm^2 = 0$

$m^2 = 0$

$\therefore m = \pm 0$

$y = A + Bx$

Obtaining the particular integral, PI

$y = Px^2 + Qx^3 + Rx^4$

$\frac{dy}{\delta x} = 2Px + 3Qx^2 + 4Rx^3$ , $\frac{\delta^2 y}{\delta x^2} = 2P + 6Qx + 12Rx^2$

$EI[2P + 6Qx + 12Rx^2] = \frac{w}{2}(L-x)^2$

$2PEI + 6QEIx + 12REIx^2 = \frac{w}{2}(L-x)^2$

$4PEI + 12QEIx + 24REIx^2 = w(L-x)^2 = w(L^2 - 2xL^2 + x^2)$

by comparing co-efficients;

$4PEI + 12QEIx + 24REIx^2 = wL^2 - 2xLw + wx^2$

$\therefore$ $24REI = w$ —(i)

$12QEI = -2Lw$ —(ii)

$4PEI = wL^2$ —(iii)

from (i), $R = \frac{w}{24EI}$ , from (ii), $Q = -\frac{Lw}{6EI}$ , from (iii), $P = \frac{wL^2}{4EI}$

$\therefore$ $y = \frac{wL^2}{4EI}x^2 - \frac{Lw}{6EI}x^3 + \frac{w}{24EI}x^4$

finding the LCM,

$y = \frac{6wL^2x^2 - 4Lwx^3 + wx^4}{24EI}$ — PI

Obtaining the general solution, G.S

$$y = A + Bx + \frac{w}{24EI}\left[6L^2x^2 - 4Lx^3 + x^4\right]$$

at $y = 0$, $x = 0$ and $\frac{dy}{dx} = 0$

$$0 = A + \cancel{\frac{w}{24EI}} \quad \therefore A = 0$$

$$\frac{dy}{dx} = B + \frac{w}{24EI}\left(12L^2x - 12Lx^2 + 4x^3\right)$$

$$0 = B \quad \therefore B = 0$$

$\therefore$ GS $= 0$

But particular solution $=$ PI $+$ GS

$$y = \frac{w}{24EI}\left(6L^2x^2 - 4Lx^3 + x^4\right)$$

$$y = \frac{wx^2}{24EI}\left(6L^2 - 4Lx + x^2\right)$$

At $x = L$,

$$y = \frac{wL^2}{24EI}\left(6L^2 - 4L^2 + L^2\right) = \frac{wL^2}{24EI}\left(3L^2\right) = \frac{3wL^4}{24EI}$$

$$\therefore \cancel{y =} \quad \therefore y = \frac{wL^4}{8EI}.$$