1a) $\lim_{x \to \frac{\pi}{2}} \left(\frac{(x^2 - \pi/4) \sin(\cos x)}{x = \pi/2} \right)$

using l'hopitals rule

$$\lim_{x \to \pi/2} \left[\frac{f(x)}{g(x)} \right] = \lim_{x \to \pi/2} \left[\frac{2 \sin(\cos x) + x^2 - \pi/4 \left[\cos(\cos x)(-\sin x) \right.}{1} \right]$$

$= 0$ //

b) $\lim_{x \to 4} \left(\frac{x^2 - 8x + 16}{x^2 - 5x + 4} \right)$

$dy/dx = \frac{2x - 8}{2x - 5} = \frac{2(4) - 8}{2(4) - 5} = \frac{0}{3} = 0$ //

c) $\lim_{x \to 2+\sqrt{3}} \cos\left(\sin^{-1}\left(\frac{x-2}{x-\sqrt{3}}\right)\right)$

$= \cos\left(\sin^{-1}\left(\frac{2+\sqrt{3}-2}{2+\sqrt{3}-\sqrt{3}}\right)\right)$

$= \cos\left(\sin^{-1}\left(\sqrt{3}/2\right)\right)$

$= \cos 60$

$= 0.5$ //

d) $\lim_{x \to \frac{\pi}{2}} \ln\left(\exp\left[\frac{3x^2 + 2x - 1}{x + 1}\right]\right)$

$\ln\left(\exp\left[\frac{3\left(\frac{\pi}{2}\right)^2 + 2\left(\frac{\pi}{2}\right) - 1}{\left(\frac{\pi}{2}\right) + 1}\right]\right)$ ×

$= 3.71$

2a) $\frac{2}{2\times 3} + \frac{2}{3\times 4} + \frac{2}{4\times 5} + \frac{2}{5\times 6} + \cdots + \frac{2}{(n+1)(n+2)}$

$U_n = \frac{2}{(n+1)(n+2)}$

$U_{n+1} = \frac{2}{(n+2)(n+3)}$

$\therefore \frac{U_{n+1}}{U_n} = \left(\frac{2}{(n+1)(n+2)}\right)^{-1} \cdot \frac{2}{(n+2)(n+3)} = \frac{n+1}{n+3}$

$\lim_{n \to \infty} = \frac{1 + \frac{1}{n}}{1 + \frac{3}{n}} = \frac{1+0}{1+0} = 1$

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Test II

$$\lim_{n\to\infty} = U_n = \lim_{n\to\infty} \frac{2}{n^2+3n+2}$$

$$= \frac{2}{0+0+2} = 1 \quad \therefore \text{ It converges}$$

2b) $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \cdots\cdots \frac{2}{n^2}$

$$U_n = \frac{2}{n^2}$$

$$U_{n+1} = \frac{2}{(n+1)^2}$$

$$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \frac{2}{(n+1)^2} \times \frac{n^2}{2} = \frac{n^2}{n^2+2n+1}$$

$$\lim_{n\to\infty} = \frac{\frac{n^2}{n^2}}{\frac{n^2}{n^2} + \frac{2n}{n^2} + \frac{1}{n^2}} = \frac{1}{1+0+0} = 1 \,//$$

Test II

$$\lim U_n = \lim_{n\to\infty} = \frac{2}{n^2} \pm 0 \quad \therefore \text{ It may be convergent or divergent}$$

2c) $U_r = \frac{1+2n^2}{1+n^2}$

$$\lim_{n\to\infty} = \frac{1+2n^2}{1+n^2} = \frac{\frac{1}{n^2} + \frac{2n^2}{n^2}}{\frac{1}{n^2} + \frac{n^2}{n^2}} = \frac{0+2}{0+1} = 2$$

$$\therefore \lim_{n\to\infty} U_n > 1$$

$\therefore$ it is divergent

3) $\frac{x}{27} + \frac{x^2}{125} + \cdots + \frac{x^n}{(2n+1)^3}$

$$U_n = \frac{x^n}{(2n+1)^3}$$

$$U_{n+1} = \frac{x^{n+1}}{(2n+3)^3}$$

$$\therefore \lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \frac{x^{n+1}}{(2n+3)^3} * \frac{(2n+1)^3}{x^3}$$

$$= \frac{x(8n^3+12n^2+6n+1)}{8n^3+36n^2+54n+27}$$

$$= \frac{8xn^3}{n^3} + \frac{12xn^2}{n^3} + \frac{6xn}{n^3} + \frac{1}{n^3}$$

$$\lim_{x\to\infty} \frac{U_{n+1}}{U_n} = \frac{8x+0+0+0}{8+0+0+0} = \frac{8x}{8} > x$$

$$\forall x : |x| < 1$$

$\therefore \forall x$ is convergent //

4) $\lim\limits_{x \to 0} \dfrac{\sin x - \cos x}{x^3}$

$dy/dx = \dfrac{-\cos x + \sin x}{3x^2} = \dfrac{\cos 0 + \sin 0}{3(0)^2}$ = undefined
$x \to 0$

$d^2x/dy^2 = \dfrac{-\sin x + \cos x}{6x} = \dfrac{-\sin 0 + \cos 0}{6(0)}$ = undefined
$x \to 0$

$d^3x/dy^3 = -\dfrac{\cos x - \sin x}{6} = \dfrac{-\cos 0 - \sin 0}{6} = -\dfrac{1}{6}$ //
$x \to 0$