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15/ENG07/1046
PETROLEUM ENGINEERING

1. $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

$y^2 - y - 2 = 0$
$y^2 - y + 2y - 2 = 0$
$y(y-1) + 2(y-1) = 0$
$\therefore y_1 = 1 \;\&\; y_2 = -2$

$y = Ae^{x} + Be^{-2x}$

Solution

Assume homogenity

$y'' - y' - 2y = 0$
$y = e^{kx}$
$y' = ke^{kx}$
$y'' = k^2e^{kx}$
$\therefore k^2e^{kx} - ke^{kx} - 2e^{kx} = 0$
$[k^2 - k - 2]e^{kx} = 0$
$k^2 - k - 2 = 0$
$k^2 - 2k + k - 2 = 0$
$(k^2 - 2k) + (k - 2) = 0$
$k(k-2) + 1(k-2) = 0$
$(k+1)(k-2) = 0$
$k + 1 = 0$
$k - 2 = 0$
$k_1 = -1 \;\&\; k_2 = 2$
$y_1 = e^{k_1x} = e^{-x}$
$y_2 = e^{k_2x} = e^{2x}$
$y = C_1Y_1 + C_2Y_2$
$y = C_1xe^{-x} + C_2xe^{2x}$
$y = C_1e^{-x} + C_2e^{2x}$
$y' = 0$
$y'' = 0$
$y_p = C$

Substitute into $y'' - y' - 2y = 8$
$0 - 0 - 2C = 8$
$-2C = 8$
$C = -4$
$\therefore y = C_1e^{-x} + C_2e^{2x} + C$

$y = C_1e^{-x} + C_2e^{2x} - 4$

2. $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

Solution

$y'' - 4y = 0$
$y = e^{kx}$
$y' = ke^{kx}$
$y'' = k^2e^{kx}$
$k^2e^{kx} - 4e^{kx} = 0$
$e^{kx}(k^2 - 4) = 0$
$k^2 - 4 = 0$
$k^2 = 4$
$k = \sqrt{4}$
$\therefore k_1 = +2 \;\&\; k_2 = -2$
$y_1 = e^{k_1x} = e^{2x}$
$y_2 = e^{k_2x} = e^{-2x}$
$y = C_1Y_1 + C_2Y_2$
$y = C_1xe^{2x} + C_2xe^{-2x}$
$y = C_1e^{2x} + C_2e^{-2x}$
$Y_p = Ae^{3x}$
$Y' = 3Ae^{3x}$
$y'' = 9Ae^{3x}$

Substitute into $y'' - y' - 4Y = 10e^{3x}$
$9Ae^{3x} - 4Ae^{3x} = 10e^{3x}$
$e^{3x}[9A - 4A] = 10e^{3x}$
$\frac{\cancel{5}A}{\cancel{5}} = \frac{10}{\cancel{5}}$
$A = 2$
$y = C_1e^{2x} + C_2e^{-2x} + Ae^{3x}$
$y = C_1e^{2x} + C_2e^{-2x} + 2e^{3x}$

3] $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

Solution

$y'' + 2y' + y = 0$

when $y'' = k^2 e^{kx}$, $y' = ke^{kx}$

$y'' = k^2 e^{kx}$

$\therefore k^2 e^{kx} + 2ke^{kx} + e^{kx} = 0$

$[k^2 + 2k + 1]e^{kx} = 0$

$k^2 + 2k + 1 = 0$

$(k^2 + k) + (k + 1) = 0$

$k(k+1) + 1(k+1) = 0$

$k + 1 = 0$

$\therefore k_1 = -1$ & $k_2 = -1$

$y_1 = e^{k_1 x} = e^{-x}$

$y_2 = e^{k_2 x} = e^{-x}$

$y = C_1 y_1 + C_2 y_2$

$y = C_1 e^{-x} + C_2 e^{-x}$

$y_p = Ae^{-2x}$

$y' = -2Ae^{-2x}$

$\therefore y'' = 4Ae^{-2x}$

Substitute into equation

$4Ae^{-2x} + 2[-2Ae^{-2x}] + Ae^{-2x} = e^{-2x}$

$\cancel{4Ae^{-2x}} - \cancel{4Ae^{-2x}} + \cancel{Ae^{-2x}} = \cancel{e^{-2x}}$

$\therefore A = 1$

$y = C_1 e^{-x} + C_2 e^{-x} + e^{-2x}$

$y = e^{-x}(C_1 + C_2) + e^{-2x}$

4] $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

Solution

$y'' + 25y = 0$

$y = e^{kx}$, $y' = ke^{kx}$, $y'' = k^2 e^{kx}$

$k^2 e^{kx} + 25e^{kx} = 0$

$e^{kx}(k^2 + 25) = 0$

$k^2 + 25 = 0$

$k^2 = -25$

$k = \pm\sqrt{-25}$

$k_1 = 5i$ & $k_2 = -5i$

$y_1 = e^{k_1 x} = e^{5ix}$

$y_2 = e^{k_2 x} = e^{-5ix}$

$y = C_1 y_1 + C_2 y_2$

$y_c = C_1 e^{5ix} + C_2 e^{-5ix}$

$y_p = Ax^2 + Bx + C$

$y' = 2Ax + B$

$y'' = 2A$

Substitute into $y'' + 25y = 5x^2 + x$

$2A + 25(Ax^2 + Bx + C) = 5x^2 + x$

$2A + 25Ax^2 + 25Bx + 25C = 5x^2 + x$

$25Ax^2 = 5x^2$

$25A = 5$

$A = 1/5$

$25Bx = x$

$25B = 1$

$B = 1/25$

$2A + 25C = 0$

$2(1/5) + 25C = 0$

$25C = -2/5$

$C = -2/125$

$\therefore y_p = Ax^2 + Bx + C = \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

$y = C_1 e^{5ix} + C_2 e^{-5ix} + \frac{1}{125}(25x^2 + 5x - 2)$

$y = C_1 \cos 5x + C_2 \sin 5x + \frac{1}{125}(25x^2 + 5x - 2)$

5) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

Solution

$y'' - 2y' + y = 0$

$y = e^{kx}$

$y' = ke^{kx}$

$y'' = k^2e^{kx}$

$k^2e^{kx} - 2ke^{kx} + e^{kx} = 0$

$[k^2 - 2k + 1]e^{kx} = 0$

$k^2 - 2k + 1 = 0$

$(k^2 - k) - (k + 1) = 0$

$k((k-1) - 1(k-1) = 0$

$k - 1 = 0$

$\therefore k_1 = 1$ & $k_2 = 1$

$y_1 = e^{k_1x} = e^x$

$y_2 = e^{k_2x} = e^x$

$y = c_1y_1 + c_2y_2$

$= c_1e^x + c_2e^x$

$y_p = A\sin x + B\cos x$

$y_p' = A\cos x - B\sin x$

$y'' = -A\sin x - B\cos x$

Substitute

$-A\sin x - B\cos x - 2(A\cos x - B\sin x) + A\sin x$
$+ B\cos x = 4\sin x$

$-A\sin x - B\cos x - 2A\cos x - 2B\sin x +$
$A\sin x + B\cos x = 4\sin x$

$(\cancel{A} + 2B \cancel{+A})\sin x + (\cancel{B} - 2A \cancel{-B})\cos x$
$= 4\sin x$

~~2B sin x = 2A cos x~~

$2B\sin x = 4\sin x$

$B = 2$

$-2A\cos x = 0\cos x$

$A = 0$

$y = c_1e^x + c_2e^x + y_p$

$y = c_1e^x + c_2e^x + A\sin x + B\cos x$

$y = c_1e^x + c_2e^x + 0\sin x + 2\cos x$

$y = e^x(c_1 + c_2) + 2\cos x$

6) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$

Solution

$y'' + 4y' + 5y = 0$

$y = e^{kx}$

$y = ke^{kx}$

$y = k^2e^{kx}$

$k^2e^{kx} + 4ke^{kx} + 5e^x = 0$

$[k^2 + 4k + 5]e^x = 0$

$k^2 + 4k + 5 = 0$

Using $k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

where $a = 1$, $b = 4$, & $c = 5$

$k = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times 5}}{2 \times 1}$

$k = \frac{-4 \pm \sqrt{16 - 20}}{2}$

$k = \frac{-4 \pm \sqrt{-4}}{2}$

$\therefore k_1 = \frac{-4 + 2i}{2} = \frac{2(-2 + i)}{2}$

$\therefore k_1 = -2 + i \quad k_2 = -2 - i$

$y_1 = e^{(-2+i)x}$

$y_2 = e^{(-2-i)x}$

$\Rightarrow y_1 = e^{-2x} \cdot e^{ix}$

$y_2 = e^{-2x} \cdot e^{-ix}$

$y = c_1y_1 + c_2y_2$

$y = c_1(e^{-2x} \cdot e^{ix}) + c_2(e^{-2x} e^{-ix})$

$y = e^{-2x}[C_1 e^{ix} + C_2 e^{-ix}]$

$y = e^{-2x}[C_1 \cos x + C_2 \sin x]$

$y_p = Ae^{-2x}$

$y' = -2Ae^{-2x}$

$y'' = 4Ae^{-2x}$

Substitute

$4Ae^{-2x} + 4(-2Ae^{-2x}) + 5(Ae^{-2x}) = 2e^{-2x}$

$4Ae^{-2x} - 8Ae^{-2x} + 5Ae^{-2x} = 2e^{-2x}$

$e^{-2x}(4A - 8A + 5A) = 2e^{-2x}$

$\cancel{e^{-2x}}(A) = 2\cancel{e^{-2x}}$

$A = 2$

$\therefore y = e^{-2x}[C_1 \cos x + C_2 \sin x] + 2e^{-2x}$

at $x = 0$, $y = 1$

$1 = e^{-2(0)}[C_1 \cos(0) + C_2 \sin(0)] + 2e^{-2(0)}$

$1 = 1(C_1 + 0) + 2$

$1 = C_1 + 2$

$C_1 = -1$

To get C_2

$y' = -2e^{-2x}[-C_1 \sin x + C_2 \cos x]$
$- 4e^{-2x}$

at $x = 0$, $y' = -2$

$-2 = -2e^{-2(0)}[-C_1 \sin(0) + C_2 \cos(0)]$
$- 4e^{-2(0)}$

$-2 = -2[0 + C_2] - 4$

$-2 = -2C_2 - 4$

$-2C_2 = 2$

$C_2 = -1$

$y = e^{-2x}[C_1 \cos x + C_2 \sin x]$
$+ 2e^{-2x}$

Substitute $C_1 = -1$ & $C_2 = -1$

$y = e^{-2x}[\cos x - \sin x] + 2e^{-2x}$

$y = e^{-2x}[-\cos x - \sin x + 2]$

$y = e^{-2x}[2 - \cos x - \sin x]$

7) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

Solution

$3y'' + 2y' - y = 0$

$y = e^{kx}$

$y' = ke^{kx}$

$y'' = k^2 e^{kx}$

$3k^2 e^{kx} - 2ke^{kx} - e^{kx} = 0$

$e^{kx}(3k^2 - 2k - 1) = 0$

$3k^2 - 2k - 1 = 0$

$(3k^2 - 3k) + (k - 1) = 0$

$3k(k - 1) + 1(k - 1) = 0$

$(3k + 1)(k - 1) = 0$

$3k + 1 = 0$ and $k - 1 = 0$

$3k = -1$ $\quad k_2 = 1$

$k_1 = -1/3$

$y_1 = e^{-1/3x}$

$y_2 = e^{x}$

$y = C_1 y_1 + C_2 y_2$

$y = C_1 e^{-1/3x} + C_2 e^{x}$

$y_p = Ax - Bx^2$

$y' = A - 0 + 0$

$y'' = 0$

Substitute

$0 - 2A - Ax - B = 2x - 3$

$-Ax = 2x$

$A = -2$ $\quad$ At $A = -2$,

$-2A - B = -3$

$-2(-2) - B = -3$

$4 - B = -3$

$B = 7$

$y_p = Ax - B = -2x + 7$

$y = C_1 e^{-1/3x} + C_2 e^{x} + y_p$

$\therefore y = C_1 e^{-1/3x} + C_2 e^{x} - 2x + 7$

8 $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

Solution

$y'' - 6y' + 8y = 0$

$y = e^{kx}$

$y' = ke^{kx}$

$y'' = k^2 e^{kx}$

$k^2 e^{kx} - 6ke^{kx} + 8e^{kx} = 0$

$[k^2 - 6k + 8]e^{kx} = 0$

$k^2 - 6k + 8 = 0$

$(k^2 - 2k) - (4k + 8) = 0$

$k(k-2) - 4(k-2) = 0$

$(k-4)(k-2)$

$k - 4 = 0$ and $k - 2 = 0$

$k_1 = 4 \quad \therefore k_2 = 2$

$y_1 = e^{k_1 x} = e^{4x}$

$y_2 = e^{2x}$

$y = C_1 y_1 + C_2 y_2$

$\therefore y = C_1 e^{4x} + C_2 e^{2x}$

$y_p = Axe^{4x}$

$y' = A[x \times 4e^{4x} + e^{4x}(1)]$

$y' = A[4xe^{4x} + e^{4x}]$

$y' = 4Axe^{4x} + Ae^{4x}$

$y'' = A[4x \cdot 4e^{4x} + e^{4x}(4) + 4e^{4x}]$

$y'' = A[16xe^{4x} + 8e^{4x}]$

$y'' = 16Axe^{4x} + 8Ae^{4x}$

Substituting

$16Axe^{4x} + 8Ae^{4x} - 6(4Axe^{4x} + Ae^{4x}) + 8(Axe^{4x}) = 8e^{4x}$

~~$8Ae^{4x} - 6Ae^{4x} = 8e$~~

$16Axe^{4x} + 8Ae^{4x} - 24Axe^{4x} - 6Ae^{4x} + 8Axe^{4x} = 8e^{4x}$

$8Ae^{4x} - 6Ae^{4x} = 8e^{4x}$

$2A\cancel{e^{4x}} = 8\cancel{e^{4x}}$

$2A = 8$

$A = 4$

$\therefore y = C_1 e^{4x} + C_2 e^{2x} + 4xe^{4x}$