NWANKWO NWACHUKWU Elect/Elect 200L 16/ENG04/036

1a) $$\lim_{x \to \frac{\pi}{2}} \left(\frac{(x^2 - \pi/4)\sin(\cos x)}{x - \pi/2} \right)$$

$$= \left(\frac{\left(\frac{\pi}{2}^2 - \frac{\pi}{4}\right)\sin\left(\cos\frac{\pi}{2}\right)}{\frac{\pi}{2} - \frac{\pi}{2}} \right) = \frac{0}{0} = \text{undefined}$$

$\therefore$ L'hopitals rule

$$\lim_{x \to \frac{\pi}{2}} \left[\frac{f(x)}{g(x)} \right] = \lim_{x \to \frac{\pi}{2}} \left[f \right.$$

$$\lim_{x \to \frac{\pi}{2}} \frac{2x\sin(\cos x) + x^2 - \pi/4\,[\cos(\cos x)(-\sin x)]}{1}$$

$$= 2x\sin\left(\cos\frac{\pi}{2}\right) + \frac{\pi}{2}^2 - \frac{\pi}{4}\left[\cos\left(\cos\frac{\pi}{2}\right)\left(-\sin\frac{\pi}{2}\right)\right]$$

$$= \underline{\underline{0}}$$

b) $$\lim_{x \to 4} \left(\frac{x^2 - 8x + 16}{x^2 - 5x + 4} \right)$$

$$= \frac{4^2 - 8(4) + 16}{4^2 - 5(4) + 4} = \frac{16 - 32 + 16}{16 - 20 + 4} = \frac{-32}{0} = \text{undefined}$$

$$\therefore \frac{dy}{dx} = \frac{2x - 8}{2x - 5}$$

$x \to 4$

$$= \frac{2(4) - 8}{2(4) - 5} = \frac{8 - 8}{8 - 5} = \frac{0}{3} = \underline{\underline{0}}$$

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2) Determine whether each of the following series is convergent

$$\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6} + \cdots + \frac{2}{(n+1)(n+2)}$$

$$u_n = \frac{2}{(n+1)(n+2)}$$

$$u_{n+1} = \frac{2}{(n+2)(n+3)}$$

$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \frac{\cancel{2}}{(n+1)\cancel{(n+2)}} \times \frac{\cancel{(n+2)}(n+3)}{\cancel{2}} = \frac{n+3}{n+1}$$

$$\therefore \lim_{n\to\infty} = \frac{n+3}{n+1}$$

$$= \frac{\frac{\cancel{n}}{\cancel{n}} + \frac{3}{n}}{\frac{\cancel{n}}{\cancel{n}} + \frac{1}{n}} = \frac{1 + \frac{3}{n}}{1 + \frac{1}{n}}$$

note when $n \to \infty$, $\frac{1}{n} \to 0$

$$\therefore \frac{1+0}{1+0} = \underline{\underline{1}}$$

Test further

$$\lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{2}{n^2+3n+2}$$

note $n \to \infty$

$\frac{1}{n} \to 0$

$$= \therefore \frac{\frac{2}{n^2}}{\frac{\cancel{n^2}}{\cancel{n^2}} + \frac{3\cancel{n}}{n^{\cancel{2}}} + \frac{2}{n^2}} = \frac{0}{1+0+0} = \underline{\underline{0}}$$

it converges or diverge

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2(c) $u_n = \dfrac{1+2n^2}{1+n^2}$

$\lim_{n\to\infty} u_n$

$= \lim_{n\to\infty} \dfrac{1+2n^2}{1+n^2}$ note $n \to \infty$, $\frac{1}{n} \to 0$

$= \therefore \dfrac{\frac{1}{n^2} + \frac{2\cancel{n^2}}{\cancel{n^2}}}{\frac{1}{n^2} + \frac{\cancel{n^2}}{\cancel{n^2}}} = \dfrac{0+2}{0+1} = \underline{\underline{2}}$

$\therefore$ It diverges

~~3)~~

2(b) $\dfrac{2}{1^2} + \dfrac{2}{2^2} + \dfrac{2}{3^2} + \dfrac{2}{4^2} \cdots \dfrac{2}{n^2}$

$u_n = 2/n^2$

$u_{n+1} = 2/(n+1)^2$

$\lim_{n\to\infty} \dfrac{u_{n+1}}{u_n} = \dfrac{\cancel{2}}{(n+1)^2} \times \dfrac{n^2}{\cancel{2}} = \dfrac{n^2}{n^2+2n+1}$

note $n \to \infty$, $\frac{1}{n} \to 0$

$\therefore = \dfrac{\frac{\cancel{n^2}}{\cancel{n^2}}}{\frac{\cancel{n^2}}{\cancel{n^2}} + \frac{2\cancel{n}}{n^{\cancel{2}}} + \frac{1}{n^2}} = \dfrac{1}{1+0+0} = \underline{\underline{1}}$

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further test

$\lim_{n\to\infty} u_n = \lim_{n\to\infty} \dfrac{2}{n^2} = \underline{\underline{0}}$ $\therefore$ It maybe convergent or divergent

C) $\lim_{x \to 2+\sqrt{3}} \cos\left(\sin^{-1}\left(\frac{x-2}{x-\sqrt{3}}\right)\right)$

$= \cos\left(\sin^{-1}\left(\frac{\cancel{2}+\sqrt{3}\cancel{-2}}{2+\cancel{\sqrt{3}}-\cancel{\sqrt{3}}}\right)\right)$

$= \cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$

$= \cos 60$

$= 1/2 = \underline{\underline{0.5}}$

D) $\lim_{x \to \frac{\pi}{2}} \ln\left(\exp\left(\frac{3x^2 + 2x - 1}{x+1}\right)\right)$

$\ln\left(\exp\left(\frac{3\left(\frac{\pi}{2}\right)^2 + 2\left(\frac{\pi}{2}\right) - 1}{\left(\frac{\pi}{2}\right)+1}\right)\right)$

$= \underline{\underline{3.71}}$

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3) $\frac{x}{27} + \frac{x^2}{125} + \cdots + \frac{x^n}{(2n+1)^3}$

$\lim_{n \to \infty} u_n = \frac{x^n}{(2n+1)^3}$

$u_{n+1} = \frac{x^{n+1}}{(2n+3)^3}$

$\therefore \lim_{n \to \infty} \frac{u_{n+1}}{u_n} = \frac{x^{n+1}}{(2n+3)^3} \times \frac{(2n+1)^3}{x^n}$

$= \frac{\cancel{x^n} \times x^1}{(2n+3)^3} \times \frac{(2n+1)^3}{\cancel{x^n}}$

$= \frac{x(2n+1)^3}{(2n+3)^3}$

$= \frac{x(8n^3 + 12n^2 + 6n + 1)}{8n^3 + 36n^2 + 54n + 27}$

$= \frac{8xn^3 + 12xn^2 + 6xn + 1}{8n^3 + 36n^2 + 54n + 2}$

$= \dfrac{\frac{8xn^3}{n^3} + \frac{12xn^2}{n^3} + \frac{6xn}{n^3} + \frac{1}{n^3}}{\frac{8n^3}{n^3} + \frac{36n^2}{n^3} + \frac{54n}{n^3} + \frac{2}{n^3}}$

$\lim_{n \to \infty} \frac{u_{n+1}}{u_n} = \therefore \frac{8x + 0 + 0 + 0}{8 + 0 + 0 + 0}$

note $n \to \infty$

$\frac{1}{n} \to 0$ $= \frac{8x}{8} = x$

$\therefore \forall x : |x| < 1$ the series converges absolutely

4) Evaluate using L'hopitals rule

$$\lim_{x \to 0} \frac{\sin x - \cos x}{x^3}$$

$$\therefore \lim_{x \to 0} \frac{dy}{dx} = \frac{\cos x + \sin x}{3x^2} = \frac{\cos 0 + \sin 0}{3(0)^2} = \text{undefined}$$

$$\therefore \lim_{x \to 0} \frac{d^2y}{dx^2} = \frac{-\sin x + \cos x}{6x} = \frac{-\sin 0 + \cos 0}{6(0)} = \text{undefined}$$

$$\therefore \lim_{x \to 0} \frac{d^3y}{dx^3} = \frac{-\cos x - \sin x}{6} = \frac{-\cos 0 - \sin 0}{6} = \frac{-1}{6}$$