NAME: ALAOMA .G. CHISOM

MATRIC NO: 15/SCI01/007

DEPT: COMPUTER ENGINEERING

COURES: ENG 381

ASSIGNMENT 3

ALAOMA · G · CHISOM
15/SCI01/007
COMP ENG

ENG 381

Assignment

(1) $y = e^{x^3+x}$; Show that : $y'' = y'(2x+1) + 2y$

Solution

$y'' = y'(2x+1) + 2y$

$y'' = \frac{d^2y}{dx^2}$ $y' = \frac{dy}{dx}$

$\therefore \frac{dy}{dx} = (2x+1)e^{x^2+x}$

using product rule $\Rightarrow V\,{}^{du}/_{dx} + U\,{}^{dv}/_{dx}$

$\frac{d^2y}{dx^2} = (2x+1)(2x+1)e^{x^2+x} + 2e^{x^2+x}$

$= (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$

$= (4x^2+4x+1)e^{x^2+x} + 2e^{x^2+x}$

$= (4x^2+4x+1+2)e^{x^2+x}$

$= (4x^2+4x+3)e^{x^2+2x}$

substituting RHS and LHS of above eqn

$(4x^2+4x+3)e^{x^2+x} = (2x+1)e^{x^2+x}(2x+1) + 2(e^{x^2+x})$.

$= (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$

$= (4x^2+4x+4)e^{x^2+x}$

$(4x^2+4x+3)e^{x^2+x} = (4x^2+4x+3)e^{x^2+x}$

$\therefore y'' = y'(2x+1) + 2y$

(ii) $y'' = y'(2x+1) + 2y$

$y'' - y'(2x+1) - 2y = 0$

Let $W = y''$

$V = 1$ $V' = 0$

$U = y$ $U^n = y^{n+2}$

$W^n = U^n V + n U^{n-1} V'$

$= y^{(n+2)} + 0$

Let $w = -y'(2x+1)$

$V = 2x+1 \qquad V' = 2 \qquad V'' = 0$

$u = -y' \qquad u^n = -y^{n+1}$

$$w^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v''$$

$= -y^{n+1}(2x+1) + n(-y^{n+1})(2) + 0$

$= -y^{n+1}(2x+1) + 2n(-y^n)$

Let, $w = -2y$

$V = -2 \qquad V' = 0$

$u = y \qquad u^n = y^n$

$w^n = u^n v + n u^{n-1} v'$

$= y^n \cdot -2 + 0$

$= -2y^n$

$\therefore \; y^n = y^{n+2} - y^{n+1}(2x+1) + 2n(-y^n) - 2y^n$

$y^{n+2} - y^{n+1}(2x+1) + 2n(-y^n) - 2y^n = 0$

$y^{n+2} - y^{n+1}(2x+1) - 2y^n(n+1) = 0$

$y^{(n+2)} = y^{(n+1)}(2x+1) + 2y^n(n-1)$

(2i) $y = x^3 e^{4x}$; find y^5

$V = x^3, \; V' = 3x^2, \; V'' = 6x, \; V''' = 6, \; V^4 =$

$u = e^{4x}, \; u' = 4e^{4x}, \; u^2 = 16e^{4x}, \; u^3 = 64e^{4x},$

$u^5 = 1024 e^{4x}$

$$y^n = u^n v + n v^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{3!} u^{n-3} v^3 + \frac{n(n-1)(n-2)(n-3)}{4!} u^{n-4} V^4$$

$$y^5 = u^5 v + 5u^4 v + \frac{5(4)}{2\times1} u^3 v^2 + \frac{5(4)(3)}{3\times2\times1} u^2 v^3 + 0$$

$y^5 = 1024 e^{4x}(x^3) + 15x^2(256 e^{4x}) + 60x(64e^{4x}) + 60(16e^{4x})$

$x^3 1024 e^{4x} + x^2 3840 e^{4x} + x 3840 e^{4x} + 960 e^{4x}$

$y^3 = 64 e^{4x} (16 x^3 + 60 x^2 + 60 x + 15)$

(2ii) $x^2 \frac{dy}{dx} - x \frac{dy}{\delta x} + y = 0$

$x^2 y'' + xy' + y = 0$

Let $w = x^2 y''$

$v = x^2$, $v' = 2x$, $v'' = 2$, $v''' = 0$

$u = y''$ $u^n = y^{n+2}$

$W^n = u^n v + n u^{n+1} v' + \frac{n(n-1)}{2!} u^{n-1} v'' + \frac{n(n-1)(n-2)}{3!} u^{n-2} v'''$

$= y^{n+2} (x^2) + n (y^{n+2})(2x) + \frac{n(n-1)}{2} (y^{n+2-1})(2)$

$= x^2 y^{n+2} + 2xn (y^{n+1}) + n(n-1)(y^n)$

Let $W = xy'$

$V = x$ $V' = 1$ $V'' = 0$

$u = y'$ $u^2 = y^{n+1}$

$W^n = y^{n+1}(x) + n(y^{n+1-1})(1) + 0$

$= xy^{n+1} + ny^n$

Let $w = y$

$V = 1$ $V' = 0$

$u = y$, $u^n = y^n$

$W = y^n$

$y^n = x^2 y^{(n+2)} + 2xn (y^{n+1}) + n(n-1)(y^n) + xy^{n+1}$
$+ y^n + y'$

$x^2 y (n-2) + (2n+1) x y^{(n+1)} + (n^2 - n + n + 1) y^n = 0$

$\therefore x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^{(n)} = 0$