

Name: Patrick Eneke Obianuzur

Matric No: 15/SC114/018

Dept: Chemical Engineering

Course code: ENG 381

① If $y = e^{x^2+x}$,

show that

$$y'' = y'(2x+1) + 2y$$

and, hence, prove that

$$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$$

Soln

$$y' = (2x+1)e^{x^2+x}$$

$$y'' = (2x+1)(2x+1)e^{x^2+x} + 2e^{x^2+x}$$

$$\underbrace{y''}_{w_1} = \underbrace{y'(2x+1)}_{w_2} + \underbrace{2y}_{w_3}$$

For w_1

$$y^{n+2}$$

For w_2

$$u = y'$$

$$v = 2x+1$$

$$u^{(n)} = y^{(n+1)}$$

$$v' = 2$$

$$u^{(n-1)} = y^{(n)}$$

$$v'' = 0$$

$$y^{(n+1)}(2x+1) + ny^{(n)}(2)$$

For w_3

$$2y^{(0)}$$

$$2y^n$$

$$w_1 = w_2 + w_3$$

$$y^{(n+2)} = y^{(n+1)}(2x+1) + ny^{(n)}2 + 2y^n$$

$$y^{(n+2)} = (2x+1)y^{(n+1)} + 2ny^{(n)} + 2y^n$$

$$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$$

(2) $y = x^3 e^{4x}$, determine $y^{(5)}$

$$y^{(n)} \rightarrow u = e^{4x}$$

$$u^n = 4^n e^{4x}$$

$$u^{(n-1)} = 4^{n-1} e^{4x}$$

$$u^{(n-2)} = 4^{n-2} e^{4x}$$

$$u^{(n-3)} = 4^{n-3} e^{4x}$$

$$V = x^3$$

$$V^1 = 3x^2$$

$$V^2 = 6x$$

$$V^3 = 6$$

$$V^4 = 0$$

$$y^n = u^n V + nu^{(n-1)} V^1 + \frac{n(n-1)}{2!} u^{(n-2)} V^2 + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} V^3 + \frac{n(n-1)(n-2)(n-3)}{4!} u^{(n-4)} V^4 \rightarrow 0$$

$$y^n = 4^n e^{4x} x^3 + n 4^{n-1} e^{4x} 3x^2 + \frac{n(n-1)}{2} 4^{n-2} e^{4x} 6x + \frac{n(n-1)(n-2)}{3!} 4^{n-3} e^{4x} 6$$

$$y^n = 4^n e^{4x} x^3 + n 4^{n-1} e^{4x} 3x^2 + n(n-1) 4^{n-2} e^{4x} 3x$$

$$y^5 = 4^5 e^{4x} x^3 + 5 \times 4^4 e^{4x} 3x^2 + 5(5-1) 4^3 e^{4x} 3x$$

$$y^5 = 1024 x^3 e^{4x} + 3840 x^2 e^{4x} + 3840 x e^{4x}$$

2ii) $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$, show that $x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

$$\underbrace{x^2 y^{(2)}}_{w_1} + \underbrace{xy^{(1)}}_{w_2} + \underbrace{y}_{w_3} = 0$$

For w_1

$$u = y^{(2)}$$

$$v = x^2$$

$$u^{(n)} = y^{(n+2)}$$

$$v^{(1)} = 2x$$

$$u^{(n-1)} = y^{(n+1)}$$

$$v^{(2)} = 2$$

$$u^{(n-2)} = y^{(n)}$$

$$v^{(3)} = 0$$

$$= y^{(n+2)} x^2 + n y^{(n+1)} 2x + \frac{n(n-1)}{2!} y^{(n)} 2 + 0$$

For w_2

$$u = y^{(1)}$$

$$v = x$$

$$u^{(n)} = y^{(n+1)}$$

$$v^{(1)} = 1$$

$$u^{(n-1)} = y^{(n)}$$

$$v^{(2)} = 0$$

$$= y^{(n+1)} x + n y^{(n)} \cdot 1 + 0$$

For w_3

$$- y^{(n)}$$

$$x^2 y^{(n+2)} + 2xn y^{(n+1)} + n(n-1)y^n + xy^{n+1} + ny^{(n)} + y^{(n)} = 0$$

$$x^2 y^{(n+2)} + 2nxy^{(n+1)} + n(n+1)y^n + xy^{n+1} + ny^{(n)} + y^{(n)} = 0$$

$$x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + [n(n-1) + n+1]y^{(n)} = 0$$

$$x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + [n^2 - n + n + 1]y^{(n)} = 0$$

$$x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + [n^2 + 1]y^{(n)} = 0$$