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Computer Engineering
15/ENG02/041

Engineering Maths III
ENG381 Assignment

1) If $y = e^{x^2+x}$
show that
$$y'' = y'(2x+1) + 2y$$

$y = e^{x^2+x}$

$\frac{dy}{dx} = y' =$

let $u = x^2 + x$

$\frac{du}{dx} = 2x + 1$

$\therefore y = e^u$, $dy/dx = y' = dy/du \times du/dx$

$\frac{dy}{dx} = y' = e^u \times 2x+1$

$y' = (2x+1)e^u$

$y' = (2x+1)e^{x^2+x}$

$\frac{d^2y}{dx^2} = y'' =$

$y' = (2x+1)e^{x^2+x}$

using product rule

let $u = 2x+1$ $\quad$ $v = e^{x^2+x}$

$\frac{du}{dx} = 2$

using chain rule

$\frac{dv}{dx} = 2x+1\, e^{x^2+x}$

$\therefore \frac{d^2y}{dx^2} = y'' = V\, du/dx + U\, dv/dx$

$= 2(e^{x^2+x}) + 2x+1(2x+1\, e^{x^2+x})$

since, $y = e^{x^2+x}$
$y' = (2x+1)e^{x^2+x}$

$\therefore y'' = 2(y) + (2x+1)(y')$

$y'' = 2y + (2x+1)y'$

$y'' = y'(2x+1) + 2y$

(b) Hence prove that
$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

<u>Solution</u>
$y'' = y'(2x+1) + 2y$

$y'' - y'(2x+1) - 2y = 0$

using leibnitz theorem

i) y''

$u = y''$ $\quad v = 1$
$u^n = y^{n+2}$ $\quad v' = 0$

$= u^n v + nu^{n-1}v'$
$= y^{n+2}(1) + n(y^{n+1})(0)$
$= y^{n+2}$

ii) $-y'(2x+1)$

$u = y'$ $\quad v = -(2x+1)$
$u^n = y^{n+1}$ $\quad v' = -2$
$v'' = 0$

$= u^n v + nu^{n-1}v' + n(n-1)u^{n-2}v''$

$= -y^{n+1}(+2x+1) + n(y^n)(-2)$

$= -(2x+1)y^{n+1} - 2ny^n$

3) $-2y$

$u = y \qquad v = -2$

$u^n = y^n \qquad v' = 0$

$= u^n v + \cancel{n u^{n-1} v'}$

$= -2y^n$

$\therefore y^{n+2} - (2x+1)y^{n+1} - 2ny^n - 2y^n = 0$

$\therefore y^{n+2} = (2x+1)y^{n+1} + 2ny^n + 2y^n$

$y^{n+2} = (2x+1)y^{(n+1)} + 2y^n(n+1)$

$\therefore y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$.

② Using the Leibnitz theorem, given that

① $y = x^3 e^{4x}$

determine $y^{(5)}$

$v = x^3$	$u = e^{4x}$
$v' = 3x^2$	$u' = 4e^{4x}$
$v^{(2)} = 6x$	$u^{(2)} = 16e^{4x}$
$v^{(3)} = 6$	$u^{(3)} = 64e^{4x}$
$v^{(4)} = 0$	$u^{(4)} = 256e^{4x}$
$v^{(5)} = 0$	$u^{(5)} = 1024e^{4x}$

$$y^5 = u^5v + nu^4v' + \frac{n(n-1)}{2!}u^3v^2 + \frac{n(n-1)(n-2)}{3!}u^2v^3 +$$

$$\frac{n(n-1)(n-2)(n-3)}{4!}u'v^4 + \frac{n(n-1)(n-2)(n-3)(n-4)}{5!}uv^5$$

$$y^5 = \left[1024e^{4x}(x^3)\right] + \left[5(256e^{4x})3x^2\right] + \left[\frac{5\times4}{2}\times 64e^{4x}\times 6x\right]$$

$$+\left[\frac{5\times4\times3}{3\times2}\times 16e^{4x}\times 6\right] + [0]\ [0]$$

$$y^5 = 1024e^{4x}x^3 + 1280e^{4x}(3x^2) + 640e^{4x}(6x) + 160e^{4x}(6)$$

$$y^5 = 1024e^{4x}x^3 + 3840e^{4x}x^2 + 3840e^{4x}x + 960e^{4x}$$

$$y^5 = e^{4x}(1024x^3 + 3840x^2 + 3840x + 960)$$

(ii) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$x^2y'' + xy + y = 0$

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① $x^2 y''$

$u = y''$ $\quad$ $v = x^2$

y $\quad$ $u^n = y^{n+2}$ $\quad$ $v' = 2x$

$v'' = 2$

$v''' = 0$

$$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + \frac{n(n-1)(n-2)}{3!} u^{n-3} v'''$$

$$y^n = y^{n+2} x^2 + n y^{n+1}(2x) + \frac{n(n-1)}{2!} y^n (2) + 0$$

$$y^n = y^{n+2} x^2 + 2xny^{n+1} + n(n-1) y^n$$

② xy'

$u = y'$ $\quad$ $v = x$

$u^n = y^{n+1}$ $\quad$ $v' = 1$

$v'' = 0$

$$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v''$$

$$y^n = y^{n+1} x + n y^n (1) + 0$$

$$y^n = x y^{n+1} + n y^n$$

③ y

$u = y$ $\quad$ $v = 1$

$u^n = y^n$ $\quad$ $v' = 0$

$$y^n = u^n v + n u^{n-1} v'$$

$$= y^n (1) + n(y^{n-1})(0)$$

$$= y^n$$

$$y^n = y^{n+2}x^2 + ny^{n+1}2x + (n(n-1))y^n + xy^{n+1} + ny^n + y^n$$

$$y^n = x^2(y^{n+2}) + 2xn(y^{n+1}) + (n^2-n)y^n + x(y^{n+1}) + ny^n + y^n$$

$$y^n = (n^2 - n + n + 1)y^n + (2xn + x)y^{n+1} + x^2(y^{n+2})$$

$$0 = x^2y^{(n+2)} + (2xn + x)y^{n+1} + (n^2+1)y^n$$

$$0 = x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n$$

$$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$$