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ELECT - ELECT

ENG 381 ASS III

① $y = e^{x^2+x}$

show that

$y'' = y'(2x+1) + 2y$

and prove that $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

Soln

$y = e^{x^2+x}$ — ①

$y^n = a^n e^{ax}$

$y' = (2x+1)e^{x^2+x}$ — ⑪

y'' = $u = 2x+1$ $\quad V = e^{x^2+x}$

$\frac{du}{dx} = 2$ $\quad \frac{dV}{dx} = (2x+1)e^{x^2+x}$

y'' using ~~Product~~ Product rule

$y'' = u\frac{dv}{dx} + V\frac{du}{dx}$

$y'' = (2x+1)(2x+1)e^{x^2+x} + 2e^{x^2+x}$

from eq 1 and (11)

$y'' = y'(2x+1) + 2y$ — *

$W^{(n)}_T = y^{(2)} = y^{(1)}(2x+1) + 2y$

$W_1^{(n)} = y^{(2)}$

$U = y^{(2)}$ $\quad V = 1$

$U^n = y^{(2+n)}$

$W_2^{(n)} = y^{(1)}(2x+1)$

$U = y^{(1)}$ $\quad V = 2x+1$

$U^n = y^{(1+n)}$ $\quad V' = 2$

$U^{n-1} = y^{(n)}$

$W_3^{(n)} = 2y$

$U = y$ $\quad V = 2$

$U^n = y^{(n)}$

$$W_1^{(n)} = W_2^{(n)} + W_3^{(n)}$$

$$y^{n} = U^{(n)}V + nU^{(n-1)}V'$$

$$y^{(2+n)} = y^{(1+n)}\cdot(2x+1) + n(y^{(n)})\cdot 2 + y^{(n)}\cdot 2$$

$$y^{(2+n)} = (2x+1)y^{(1+n)} + 2(n+1)y^{(n)}$$

2. Using Leibnitz theorem:

(1) $y = x^3 e^{4x}$, determine $y^{(5)}$

Soln

$U = e^{4x}$	$V = x^3$
$U^{n} = 4^{n}e^{4x}$	$V' = 3x^2$
$U^{n-1} = 4^{n-1}e^{4x}$	$V^2 = 6x$
$U^{n-2} = 4^{n-2}e^{4x}$	$V^3 = 6$
$U^{n-3} = 4^{n-3}e^{4x}$	

$$y^{(n)} = U^{(n)}V + nU^{(n-1)}V' + \frac{n(n-1)U^{(n-2)}V^{(2)}}{2!} + \frac{n(n-1)(n-2)U^{(n-3)}V^{(3)}}{3!} + \cdots$$

$$y^{(5)} = 4^5e^{4x}\cdot x^3 + 5(4^4e^{4x})\cdot 3x^2 + \frac{5(4)(4^3e^{4x})}{2!}\cdot 6x + \cdots$$

$$+ \frac{5(4)(3)(4^2e^{4x})}{3!}\cdot 6$$

$$y^{(5)} = 1024x^3e^{4x} + 3840x^2e^{4x} + 3840xe^{4x} + 960e^{4x}$$

$$y^{(5)} = 64e^{4x}[16x^3 + 60x^2 + 60x + 15]$$

(2) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Show that $x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

Soln

$$x^2y^{(2)} + xy^{(1)} + y = 0$$

$W_1 = x^2y^{(2)}$		$W_2 = xy^{(1)}$		$W_3 = y$	
$U = y^{(2)}$	$V = x^2$	$U = y^{(1)}$	$V = x$	$U = y$	$V = 1$
$U^{(n)} = y^{(2+n)}$	$V' = 2x$	$U^{(n)} = y^{(1+n)}$	$V' = 1$	$U^{n} = y^{(n)}$	
$U^{(n-1)} = y^{(1+n)}$	$V^{2} = 2$	$U^{(n-1)} = y^{(n)}$			
$U^{(n-2)} = y^{(n)}$					

Using Leibnitz theorem

$$y^n = U^{(n)}V + nU^{(n-1)}V^{(1)} + \frac{n(n-1)U^{(n-2)}V^{(2)}}{2!}$$

$$w_1^{(n)} = y^{(2+n)} \cdot x^2 + n(y^{(1+n)} \cdot 2x) + \frac{n(n-1)}{\cancel{2}!} y^{(n)} \cdot \cancel{2}$$

$$w_1^{(n)} = x^2 y^{(n+2)} + 2xny^{(1+n)} + n(n-1)y^{(n)}$$

$$w_2^{(n)} = y^{(1+n)} \cdot x + ny^{(n)} \cdot 1$$

$$w_2^{(n)} = xy^{(1+n)} + ny^{(n)}$$

$$w_3^{(n)} = y^{(n)} \cdot 1 = y^{(n)}$$

adding all together

$$x^2y^{(n+2)} + 2xny^{(1+n)} + n(n-1)y^{(n)} + xy^{(1+n)} + ny^{(n)} + y^{(n)} = 0$$

$$x^2y^{(n+2)} + xy^{(1+n)}[2n+1] + y^{(n)}[n(n-1) + n + 1] = 0$$

$$x^2y^{(n+2)} + xy^{(1+n)}[2n+1] + y^{(n)}[n^2 - n + n + 1] = 0$$

$\therefore$

$$x^2y^{(n+2)} + [2n+1]xy^{(1+n)} + [n^2+1]y^{(n)} = 0$$