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Assignment

Question 1: $y = e^{x^2+x}$

show that $y'' = y'(2x+1) + 2y$ and prove that $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^{n}$

Solution

$y = e^{x^2+x}$ - - - (1)

$\therefore$ $y^n = a^n e^{ax}$

$y' = (2x+1)e^{x^2+x}$ - - - (2)

where $u\frac{dv}{dx} + v\frac{du}{dx}$

$u = 2x+1$, $\frac{du}{dx} = 2$,

$v = e^{x^2+x}$, $\frac{dv}{dx} = (2x+1)e^{x^2+x}$

$y'' = (2x+1)(2x+1)e^{x^2+x} + 2e^{x^2+x}$

from eqn (1) and (2)

$y'' - y'(2x+1) + 2y$

Let $w_1 = y''$, $w_0 = y'(2x+1)$

$u = y^{2}$, $v = 1$

$u^n = y^{(2+n)}$

$u = y^{(1)}$ $v = 2x+1$

$u^n = y^{(1+n)}$ $v' = 2$

$u^{n+1} = y^n$

$w_2 = 2y$

$u = y$ $v = 2$

$u^n = y^{(n)}$

$\therefore W_1^{(n)} = W_2^{(n)} + W_3^{(n)}$

$y^n = u^{(n)}v + nu^{(n-1)}v'$

$y^{(2+n)} = y^{(1+n)} \cdot (2x+1) + n(_1 y^{(n)}) \cdot 2 + y^{(n)} \cdot 2$

$y^{(2+n)} = (2x+1)y^{(1+n)} + 2(n+1)y^{(n)}$

Question 2:-

$y = x^3 e^{4x}$, determine $y^{(5)}$

Solution

$u = e^{4x}$ $\quad$ $v = x^3$

$u^n = 4^n e^{4x}$ $\quad$ $v' = 3x^2$

$u^{n-1} = 4^{n-1} e^{4x}$ $\quad$ $v'' = 6x$

$u^{n-2} = 4^{n-2} e^{4x}$ $\quad$ $v''' = 6$

$u^{n-3} = 4^{n-3} e^{4x}$

$$y^{(n)} = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v'' + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v'''$$

$$y^{(5)} = 4^5 e^{4x} \cdot x^3 + 5(4^4 e^{4x}) \cdot 3x^2 + \frac{5(4)}{2!}(4^3 e^{4x}) \cdot 6x + \cdots +$$

$$\frac{5(4)(3)(4^2 e^{4x})}{3!} \cdot 6$$

$$y^{(5)} = 1024x^3 + 3840x^2 e^{4x} + 3840x e^{4x} + 96e^{4x}$$

$$y^{(5)} = 64e^{4x}[16x^3 + 60x^2 + 60x + 15]$$

ii) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Show that $x^2 y^{(n+2)} + (2n+1)x y^{(n+1)} + (n^2+1)y^n = 0$

Solution

Let $w = x^2 y$

$u = y''$, $u' = y'''$, $u'' = y^{iv}$ $\cdots$ $u^{(n)} = y^{(n+2)}$

$v = x^2$

$v' = 2x$

$v'' = 2$

$v''' = 0$

$$W^n = U^n V^0 + \frac{n U^{n-1}}{1} V' + \frac{n(n-1) U^{n-2}}{1\cdot 2} V'' + \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3} U^{n-3} V''' + \dots$$

$$W^n = y^{n+2} x^2 + n \cdot y^{n+1} \cdot 2x + \frac{n(n-1)}{2} y^n \cdot 2 + \cancel{\frac{n(n-1)(n-2)}{6} y^{n-1}} \cdot 0$$

$$W^n = y^{n+2} \cdot x^n + n \cdot 2x y^{n+1} + n(n-1) y^n$$

Let $w = x y'$

$u = y'$, $u' = y''$, $u^n = y^{(n+1)}$

$v = x$

$v' = 1$

$v'' = 0$

$$W^{(n)} = U^n V^0 + \frac{n U^{n-1}}{1} V' + \frac{n(n-1) U^{n-2}}{1\cdot 2} V''$$

$$W^{(n)} = y^{n+1} + x + \frac{n y^n}{1} \cdot 1 + \cancel{\frac{n(n-1)}{1\cdot 2} y^{n-1}} \cdot 0$$

$$W^{(n)} = y^{n+1} + x + n y^n \cdot 1$$

where $w = y$

$w^n = y^n$

$\therefore\ y^{n+2} x^2 + n \cdot y^{n+1} \cdot 2x + n(n-1) y^n + y^{n+1} + x + n y^n + y^n = 0$

$\Rightarrow y^{n+2} x^2 + (n \cdot 2x y^{n+1} + x y^{n+1}) + [n(n-1) y^n + n y^n + y^n] = 0$

$\Rightarrow x^2 y^{n+2} + (2n+1) x y^{n+1} + [n(n-1) + 1 + n) y^n = 0$

$\Rightarrow x^2 y^{n+2} + (2n+1) x y^{n+1} + (n^2 - n + 1 + n) y^n = 0$

$\Rightarrow x^2 y^{n+2} + (2n+1) x y^{n+1} + (n^2+1) y^n = 0$