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1 If $y=e^{x^2+x}$, show that $y''=y'(2x+1)+2y$ and hence prove that

$$y^{(n+2)}=(2x+1)y^{(n+1)}+2(n+1)y^{n}$$

Soln

$y=e^{x^2+x}$; $u=e^{x^2+x}$; $v=1$

$u^{n}=(2x+1)^{n}e^{x^2+x}$ $\quad v^{(1)}=0$

$u^{(n-1)}=(2x+1)^{(n-1)}e^{x^2+x}$

$y^{(n)}=u^{(n)}v+nu^{(n-1)}v^{(1)}$ $\quad ;\ y^{(n)}=(2x+1)^{n}e^{x^2+x}$

let $n=1$

$y^{(1)}=(2x+1)^{1}e^{x^2+x}$ $\quad ; u=e^{x^2+x}$ $\quad ; v=2x+1$

$u^{(n)}=(2x+1)^{n}e^{x^2+x}$ $\quad v^{(1)}=2$

$u^{(n-1)}=(2x+1)^{n-1}e^{x^2+x}$ $\quad v^{(2)}=0$

$y^{1(n)}=(2x+1)^{n}e^{x^2+x}\cdot(2x+1)+n(2x+1)^{n-1}e^{x^2+x}\cdot 2$

$=(2x+1)(2x+1)^{n}e^{x^2+x}+2n(2x+1)^{n-1}e^{x^2+x}$

let $n=1$

$y''=(2x+1)(2x+1)^{1}e^{x^2+x}+2n(2x+1)^{0}e^{x^2+x}$

$y''=(2x+1)(2x+1)^{1}e^{x^2+x}+2(e^{x^2+x})$

Substitute $y=e^{x^2+x}$ and $y'=(2x+1)^{1}e^{x^2+x}$ in y''

$y''=(2x+1)y'+2y$

$$\underline{\underline{y''=y'(2x+1)+2y}}$$

This can also be written as

$y^{(2)}=y^{(1)}(2x+1)+2y$

$y^{(n)}=u^{(n)}v+nu^{(n-1)}v^{(1)}$

; $u=y^{(1)}$	$v=2x+1$	; $u=y$	; $v=2$
$u^{n}=y^{(n+1)}$	$v^{1}=2$	$u^{n}=y^{n}$	$v^{1}=0$
$u^{(n-1)}=y^{(n)}$	$v''=0$		

$y^{(n+2)}=y^{(n+1)}\cdot(2x+1)+\cdots$

$\cdots+ny^{(n)}\cdot 2+y^{n}\cdot 2$

$y^{(n+2)}=(2x+1)y^{(n+1)}+2ny^{(n)}+2y^{(n)}$

$y^{(n+2)}=(2x+1)y^{(n+1)}+2(n+1)y^{(n)}$

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2. Using the Leibnitz theorem

i. $y = x^3 e^{4x}$, determine $y^{(5)}$

$u = e^{4x}$; $v = x^3$

$u^{(n)} = 4^n e^{4x}$; $v^{(1)} = 3x^2$

$u^{(n-1)} = 4^{(n-1)} e^{4x}$; $v^{(2)} = 6x$

$u^{(n-2)} = 4^{(n-2)} e^{4x}$; $v^{(3)} = 6$

$u^{(n-3)} = 4^{(n-3)} e^{4x}$; $v^{(4)} = 0$

$$y^{(n)} = u^{(n)} v + n u^{(n-1)} v^{(1)} + \frac{n(n-1)}{2!} u^{(n-2)} v^{(2)} + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v^{(3)}$$

$$y^{(n)} = 4^n e^{4x} \cdot x^3 + n \cdot 4^{(n-1)} e^{4x} \cdot 3x^2 + \frac{n(n-1)}{2 \times 1} \cdot 4^{(n-2)} e^{4x} \cdot 6x + \frac{n(n-1)(n-2)}{3 \times 2 \times 1} \cdot 4^{(n-3)} e^{4x} \cdot 6$$

$$y^{(5)} = 4^5 e^{4x} \cdot x^3 + 5 \cdot 4^{(4)} e^{4x} \cdot 3x^2 + \frac{5 \times 4}{2 \times 1} \cdot 4^3 e^{4x} \cdot 6x + \frac{5 \times 4 \times 3}{3 \times 2 \times 1} \cdot 4^2 e^{4x} \cdot 6$$

$$y^{(5)} = 1024 x^3 e^{4x} + 3840 x^2 e^{4x} + 3840 x e^{4x} + 960 e^{4x}$$

$$y^{(5)} = 64 e^{4x} (16x^3 + 60x^2 + 60x + 15)$$

ii. $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$, show that $x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^{(n)} = 0$

$x^2 y'' + x y' + y = 0 \Rightarrow x^2 y^{(2)} + x y^{(1)} + y^{(0)} = 0$

$u = y^{(2)}$ $v = x^2$	$u = y^{(1)}$ $v = x$	$u = y^{(0)}$ $v = 1$
$u^{(n)} = y^{(n+2)}$ $v^1 = 2x$	$u^{(n)} = y^{(n+1)}$ $v^1 = 1$	$u^{(n)} = y^{(n)}$ $v^1 = 0$
$u^{(n-1)} = y^{(n+1)}$ $v^{(2)} = 2$	$u^{(n-1)} = y^{(n)}$ $v^{(2)} = 0$	
$u^{(n-2)} = y^{(n)}$		

$$y^{(n)} = u^{(n)} v + n u^{(n-1)} v^{(1)} + \frac{n(n-1)}{2!} u^{(n-2)} v^{(2)}$$

$$y^{(n)} = y^{(n+2)} \cdot x^2 + n y^{(n+1)} \cdot 2x + \frac{n(n-1)}{\cancel{2} \times 1} y^{(n)} \cdot \cancel{2} + y^{(n+1)} \cdot x + n y^{(n)} \cdot 1 + y^{(n)} \cdot 1$$

$$y^{(n)} = x^2 y^{(n+2)} + 2xn y^{(n+1)} + n(n-1) y^{(n)} + x y^{(n+1)} + n y^{(n)} + y^{(n)}$$

$$y^{(n)} = x^2 y^{(n+2)} + 2nxy^{(n+1)} + xy^{(n+1)} + (n^2-n)y^{(n)} + ny^{(n)} + y^{(n)}$$

$$y^{(n)} = x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2-\cancel{n}) + (\cancel{n}+1)y^{(n)}$$

$$y^{(n)} = x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)}$$

$$x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$$