OLATUNJI TEMITOPE OLATUNDE
15/ENG02/043
COMPUTER ENGINEERING
ENG 381 ASSIGNMENT III

(1) $y = e^{x^2+x}$

show that

$y'' = y'(2x+1) + 2y$

and hence prove that

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^{n}$

Solution

$y = e^{x^2+x}$ ——— (i)

$y^n = a^n e^{ax}$

$y' = (2x+1)e^{x^2+x}$ ——— (ii)

$u = 2x+1$ $\quad$ $v = e^{x^2+x}$

$\frac{du}{dx} = 2$ $\quad$ $\frac{dv}{dx} = (2x+1)e^{x^2+x}$

using Product rule

$y'' = u\frac{dv}{dx} + v\frac{du}{dx}$

$y'' (2x+1)(2x+1)e^{x^2+x} + 2e^{x^2+x}$

from eqn (i) and (ii)

$y'' = y'(2x+1) + 2y$

$y^{(2)} = y^{(1)}(2x+1) + 2y$

$w_1 = y^{(2)}$ $\quad$ $w_2 = y^{(1)}(2x+1)$

$u = y^{(1)}$ $\quad$ $v = 1$ $\quad$ $u = y^{(1)}$ $\quad$ $v = 2x+1$

$u^n = y^{(2+n)}$ $\quad$ $u^n = y^{(1+n)}$ $\quad$ $v' = 2$

$u^{n-1} = y^{(n)}$

$w_3 = 2y$

$u = y$ $\quad$ $v = 2$

$u^n = y^{(n)}$

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$u = y$ $\quad$ $v = 2$

$u^n = y^{(n)}$

$$w_1^{(n)} = w_2^{(n)} + w_3^{(n)}$$

$$y^{n} = u^{(n)} v + n u^{(n-1)} v'$$

$$y^{(2+n)} = y^{(1+n)} \cdot (2x+1) + n(y^{(n)}) \cdot 2 + y^{(n)} \cdot 2$$

$$y^{(2+n)} = (2x+1) y^{(1+n)} + 2(n+1) y^{(n)}$$

2: Using Leibnitz theorem, given that

(i) $y = x^3 e^{4x}$, determine $y^{(5)}$

(ii) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$, show that

$$x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0.$$

Solution

(i) $y = x^3 e^{4x}$, determine $y^{(5)}$.

Solution

$u = e^{4x}$ $\quad v = x^3$

$u^n = 4^n e^{4x}$ $\quad v' = 3x^2$

$u^{n-1} = 4^{n-1} e^{4x}$ $\quad v'' = 6x$

$u^{n-2} = 4^{n-2} e^{4x}$ $\quad v^3 = 6$

$u^{n-3} = 4^{n-3} e^{4x}$

$$y^{(5)} = u^{(n)}v + nu^{(n-1)}v' + \frac{n(n-1)u^{(n-2)}v^{(2)}}{2!} + \frac{n(n-1)(n-2)u^{(n-3)}v^{(3)}}{3!} + \dots$$

$$y^{(5)} = 4^5 e^{4x} x^3 + 5(4^4 e^{4x}).3x^2 + \frac{5(4)(4^3 e^{4x})}{2!}.3\cancel{6x}$$

$$+ \frac{5(4)(3)(4^2 e^{4x})}{3!}.6^2$$

$$y^{(5)} = 1024x^3 e^{4x} + 3840x^2 e^{4x} + 3840xe^{4x} + 960e^{4x}$$

$$y^{(5)} = 64e^{4x}[16x^3 + 60x^2 + 60x + 15]$$

(11) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

show that $x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

Solution

$x^2y^{(2)} + xy^{(1)} + y = 0$

$w_1 = x^2y^{(2)}$

$u = y^{(2)}$ $\quad v = x^2$

$u^{(n)} = y^{(2+n)}$ $\quad v' = 2x$

$u^{(n-1)} = y^{(1+n)}$ $\quad v^2 = 2$

$u^{(n-2)} = y^{(n)}$

$w_2 = xy^{(1)}$

$u = y'$ $\quad v = x$

$u^{(n)} = y^{(1+n)}$ $\quad v' = 1$

$u^{(n-1)} = y^{(n)}$

$w_3 = y$

$u = y$ $\quad v = 1$

$u^n = y^{(n)}$

using leibnitz theorem

$y^n = u^{(n)}v + nu^{(n-1)}v^{(1)} + \frac{n(n-1)u^{(n-2)}v^{(2)}}{2!}$

$$W_1^{(n)} = y^{(2+n)} \cdot x^2 + n\left(y^{(1+n)} \cdot 2x\right) + \frac{n(n-1)y^{(n)}}{2!}$$

$$W_1(n) = x^2 y^{(n+2)} + 2xny^{(1+n)} + n(n-1)y^{(n)}$$

$$W_2(n) = y^{(1+n)} \cdot x + ny^{(n)} \cdot 1$$

$$W_2(n) = xy^{(1+n)} + ny^{(n)}$$

$$W3^{(n)} = y^{(n)} \cdot 1 = y^{(n)}$$

Solving all together

$$x^2 y^{(n+2)} + 2xny^{(1+n)} + n(n-1)y^{(n)} + xy^{(1+n)} + ny(n) + y(n) = 0$$

$$x^2 y^{(n+2)} + xy^{(1+n)}\left[2n+1\right] + y^{(n)}\left[n(n-1) + n + 1\right] = 0$$

$$x^2 y^{(n+2)} + xy^{(1+n)}\left[2n+1\right] + y^{(n)}\left[n^2 - n + n + 1\right] = 0$$

$$x^2 y^{(n+2)} + \left[2n+1\right]xy^{(1+n)} + \left[n^2+1\right]y^{(n)} = 0$$