SUNNY OGHENEFERUME COLLINS
16/ENG04/070
ELECT/ELECT ENGINEERING
ASSIGNMENT 3 ENG 381

(1) If $Y = e^{x^2+x}$

Show that

$y'' = y'(2x+1) + 2y$

hence I prove that

$y^{(n+2)} = (2x+1)\,y^{(n+1)} + 2(n+1)y^n$

Soln

$Y = e^{x^2+x}$ —— (1)

Taking the first derivative

$dy/dx = (2x+1)e^{x^2+x} \Rightarrow y'$ —— (2)

Taking the second derivative using product rule

$d^2y/dx^2 = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}(2)$

$d^2y/dx^2 = (4x^2+4x+1)e^{x^2+x} + 2e^{x^2+x} \Rightarrow y''$ —— (3)

Now equating eq (3) to eq (2) and eq (1)

$(4x^2+4x+1)e^{x^2+x} = (2x+1)e^{x^2+x}(2x+1) + 2(e^{x^2+x})$

$(4x^2+4x+1)e^{x^2+x} = (4x^2+4x+1)e^{x^2+x} + 2e^{x^2+x}$ —— (4)

from eq (4)

$\therefore\ y'' = y'(2x+1) + 2y$

(ii) $y'' = y'(2x+1) + 2y$

Using leibnitz theorem

$-y'' + y'(2x+1) + 2y = 0$

Taking the first term

$y'' \Rightarrow d^2y/dx^2$

$u = y''$, $u^n = y^{(n+2)}$

$v = 1$, $v' = 0$

$y^{(n)} = y^{(n+2)} + 0 \Rightarrow y^{(n+2)}$ —— (1)

Taking the second term

$u = y'$ $u^n = y^{(n+1)}$

$v = 2x+1$ $v' = 2$ $v'' = 0$

$y^{(n)} = y^{(n+1)}\,2x+1 + ny^{(n)}\,2 + 0$ —— (2)

Taking the third term

$u = y$ $u^n = y^{(n)}$

$v = 2$ $v' = 0$

$y^{(n)} = y^{(n)}\,2 + 0 \Rightarrow 2y^{(n)}$ —— (3)

Combining the three equations

$y^{(n+2)} = y^{(n+1)} 2x+1 + 2ny^{(n)} + 2y^{(n)}$

Collect like terms

$y^{(n+2)} = 2x+1\, y^{(n+1)} + 2y^{(n)}(n+1)$

(2i) $y = x^3 e^{4x}$, determine $y^{(5)}$

$y' = x^3(4e^{4x}) + e^{4x}(3x^2)$

$y' = x^3 4e^{4x} + 3x^2 e^{4x}$ —— (i)

$y'' = x^3(16e^{4x}) + 4e^{4x}3x^2 + 3x^2 4e^{4x} + 6xe^{4x}$

$y''' = x^3 64e^{4x} + 16e^{4x}3x^2 + 3x^2 16e^{4x} + 6x4e^{4x} + 3x^2 16e^{4x}$
$+ 6x4e^{4x} + 6x4e^{4x} + 6e^{4x}$

$y^{iv} = x^3 256e^{4x} + 3x^2 64e^{4x} + 3x^2 64e^{4x} + 6x$
$16e^{4x} + 3x^2 64e^{4x} + 6x16e^{4x} + 6x16e^{4x} + 96e^{4x}$
$+ 3x^2 64e^{4x} + 6x16e^{4x} + 6x16e^{4x} + 24e^{4x} + 6x16e^{4x}$
$+ 24e^{4x} + 24e^{4x} + 0$

$y^{v} = x^3 1024e^{4x} + 3x^2 256e^{4x} + 3x^2 256e^{4x} + 6x64e^{4x} + 3x^2 256e^{4x}$
$+ 6x64e^{4x} + 6x64e^{4x} + 96e^{4x} + 3x^2 256e^{4x} + 6x64e^{4x}$
$+ 96e^{4x} + 384e^{4x} + 0 + 3x^2 256e^{4x} + 6x64e^{4x} + 6x64e^{4x}$
$+ 96e^{4x} + 6x64e^{4x} + 96e^{4x} + 96e^{4x} + 0 + 6x64e^{4x} + 96e^{4x}$
$+ 96e^{4x} + 0 + 96e^{4x} + 0$

(ii) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Show that $x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

Soln

$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Taking the first term

$x^2 \, d^2y/dx^2 \Rightarrow x^2 y''$

$u = y''$ $\quad u^n = y^{(n+2)}$

$v = x^2$ $\quad v' = 2x$ $\quad v'' = 2$ $\quad v''' = 0$

$y^n = x^2 y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^{(n)}$ —— (1)

Taking the second term

$x\, dy/dx \Rightarrow xy'$

$u = y'$ $\quad u^n = y^{n+1}$

$v = x$ $\quad v' = 1$ $\quad v'' = 0$

$$y^n = y^{(n+1)}x + ny^{(n+1-1)} + 0$$

$$y^n = xy^{(n+1)} + ny^{(n)} \quad \text{—— (2)}$$

Taking the third term

$u = y, \quad u^n = y^n$

$v = 1 \quad v' = 0$

$y^n = y^n + 0 \Rightarrow y^n$ —— (3)

Now Combining eq (1), (2) & (3)

$$x^2y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^{(n)} + xy^{(n+1)} + ny^{(n)} + y^n = 0$$

Now Collecting like terms

$$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + y^{(n)}(n^2 - \cancel{n} + \cancel{n} + 1) = 0$$

$$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + y^{(n)}(n^2+1) = 0$$