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ENG381 - Engineering Mathematics.

1i If $y = e^{x^2+x}$ show that $y'' = y'(2x+1) + 2y$ and hence prove that
$y^{(n+2)} = (2x+1)y^{n+1} + 2(n+1)y^n$

Solution

$y = e^{x^2+x}$ ... (i)

Taking the first derivative

$\frac{dy}{dx} = y' = (2x+1)e^{x^2+x}$ ... (ii)

$\frac{d^2y}{dx^2} = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}(2)$

$\frac{d^2y}{dx^2} = y'' = (4x^2+4x+1)e^{x^2+x} + 2e^{x^2+x}$ ... (iii)

Equating (iii) to (ii) and (i)

$(4x^2+4x+1)e^{x^2+x} = (2x+1)e^{x^2+x}(2x+1) + 2(e^{x^2+x})$ ... (iv)

$(4x^2+4x+1)e^{x^2+x} = (4x^2+4x+1)e^{x^2+x} + 2e^{x^2+x}$ ... v.

Thus, $\cancel{y''} = y'$ if we observe in eqn (iv), ~~since~~.

$y'' = y'(2x+1) + 2y$.

ii $y'' = y'(2x+1) + 2y$

Using leibnitz theorem.

$-y'' + y'(2x+1) + 2y = 0$

Let $W = y''$

$W' = y'''$

$W^n = -y^{n+2}$

Also, ~~let~~

Let: $W = y'(2x+1)$

$\therefore u = y'$; $u' = y''$, $u'' = y'''$, $u^n = y^{n+1}$

$v = 2x+1$; $v' = 2$, $v'' = 0$

Also Let $P = y'(2x+1)$

$\therefore u = y'$ & $v = 2x+1$

$u' = y''$ $v' = 2$

$u'' = y'''$ $v'' = 0$

$u^n = y^{n+1}$

$$p^{(n)} = u^n v^0 + n u^{n-1} v' + n(n-1) u^{n-2} v''$$
$$= y^{n+1}(2x+1) + n y^n . 2 + 0$$
$$p^{(n)} = (2x+1) y^{n+1} + 2n y^n$$

Let $Z = 2y$
$$Z^n = 2y^n$$

Adding ; $W^n + P^n + Z^n$
$$-y^{n+2} + (2x+1) y^{n+1} + 2n y^n + 2y^n = 0$$
$$\Rightarrow y^{n+2} = (2x+1) y^{n+1} + 2(n+1) y^n$$

2

No: 2

Using leibnitz theorem, given that $y = x^3 e^{4x}$, determine $y^{(5)}$.

Solution

$$y = x^3 e^{4x}$$

Let $U = e^{4x}$ and $V = x^3$

$U^{(i)} = 4e^{4x}$ $\quad V' = 3x^2$

$U^{(ii)} = 16e^{4x}$ $\quad V'' = 6x$

$U^{(iii)} = 64e^{4x}$ $\quad V''' = 6$

$U^{(iv)} = 256e^{4x}$ $\quad V^{iv} = 0$

$U^{(v)} = 1024e^{4x}$

From Leibnitz theorem,
$$y^n = U^n V^0 + \frac{n U^{(n-1)} V'}{1!} + \frac{n(n-1) U^{(n-2)} V''}{2!} + \frac{n(n-1)(n-2) U^{(n-3)} V'''}{3!} + \frac{n(n-1)(n-2)(n-3) U^{(n-4)} V^{iv}}{4!}$$

Comparing ; $n = 5$,

~~$y^{(5)} = e^{4x} . x^3 + 5 .$~~

$$y^{(5)} = U^{(v)} V^0 + 5 \frac{U^{(iv)}}{1} . V' + \frac{(5\times4) U^{(iii)} V^{(ii)}}{2} + \frac{(5\times4\times3) U'' V'''}{6} + \frac{5\times4\times3\times2\ U' . V^{(iv)}}{8}$$

$$y^{(5)} = 1024e^{4x} . x^3 + 5(256e^{4x}) 3x^2 + 10(64e^{4x}) 6x + 10(16e^{4x}) . 6 + 0$$
$$y^{(5)} = 1024x^3 e^{4x} + 3840x^2 e^{4x} + 3840x e^{4x} + 960e^{4x}$$

(2ii)

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

Show that $x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^{(n)} = 0$

Solution

$$x^2 y'' + xy' + y = 0$$

Lets $W = x^2 y''$

So that; $U = y''$ $ $\quad$ $V = x^2$

$U' = y'''$ $\quad$ $V' = 2x$

$U'' = y^{iv}$ $\quad$ $V'' = 2$

$U^n = y^{(n+2)}$ $\quad$ $V''' = 0$

$$W^{(n)} = U^n V^0 + n\frac{U^{n-1}}{1!}V' + \frac{n(n-1)U^{n-2}V''}{2!} + \frac{n(n-1)(n-2)U^{n-3}}{3!}V'''$$

$$= y^{(n+2)} \cdot x^2 + n y^{(n+1)} \cdot 2x + \frac{n(n-1)y^n \cdot 2}{2} + 0$$

$$W^{(n)} = x^2 y^{(n+2)} + n2x y^{(n+1)} + n(n-1) y^n$$

Also, $W = xy'$

So that; $U = y'$ $\quad$ and $V = x$

$U' = y''$ $\quad$ $V' = 1$

$U^n = y^{(n+1)}$ $\quad$ $V'' = 0$

$$W^{(n)} = U^n V^0 + n U^{(n-1)} V' + \frac{n(n-1)U^{(n-2)}}{2!} V''$$

$$W^{(n)} = y^{(n+1)} \cdot x + n y^n + 0$$

$$W^{(n)} = x y^{(n+1)} + n y^n$$

~~Also~~

Also; $W = y$

$W^{(n)} = y^n$

Adding;: $x^2 y^{(n+2)} + n2x y^{(n+1)} + n(n-1) y^n + x y^{(n+1)} + n y^n + y^n = 0$

$\Rightarrow x^2 y^{(n+2)} + (2n+1) y^{(n+1)} + (n^2 - n + n + 1) y^n = 0$

$x^2 y^{(n+2)} + (2n+1) y^{(n+1)} + (n^2+1) y^n = 0$

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