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16/ENG04/063

1) $y = e^{x^2+x}$

Proving that $y'' = y'(2x+1) + 2y$

solution

$y = e^{x^2+x}$

$\frac{dy}{dx} = y' = (2x+1)e^{x^2+x}$

$\frac{d^2y}{dx^2} = y'' = 2\cdot e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$y'' = y'(2x+1) + 2y$

$2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x} = (2x+1)e^{x^2+x}(2x+1) + 2(e^{x^2+x})$

$2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x} = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$\therefore y'' = y'(2x+1) + 2y$

Proving that $y^{n+2} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

$\underset{w_1}{y''} = \underset{w_2}{y'(2x+1)} + \underset{w_3}{2y}$

$\underline{w_1}$

$U = y^{(2)}$ $\qquad V = 1$

$U^n = y^{(n+2)}$

$\underline{w_2}$

$U = y'$ $\qquad V = 2x+1$

$U^n = y^{(n+1)}$ $\qquad V' = 2$

$U^{(n-1)} = y^{(n)}$

$\underline{w_3}$

$U = y$ $\qquad V = 2$

$U^n = y^n$

$w_1 = w_2 + w_3$ $\qquad$ Applying leibnits theorem

$y^{(n+2)}\cdot 1 = y^{(n+1)}\cdot(2x+1) + ny^n\cdot 2 + y^n\cdot 2$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2ny^n + 2y^n$

$y^{(n+2)} = (2x+1)y^{(n+1)} + (2n+2)y^n$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

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(i) find $y^{(5)}$ if $y = x^3 e^{4x}$

solution

$y = x^3 e^{4x}$

$U = e^{4x}$ $\qquad V = x^3$

$U^n = 4^n e^{4x}$ $\qquad V' = 3x^2$

$U^{(n-1)} = 4^{(n-1)} e^{4x}$ $\qquad V^{(2)} = 6x$

$U^{(n-2)} = 4^{(n-2)} e^{4x}$ $\qquad V^{(3)} = 6$

$U^{(n-3)} = 4^{(n-3)} e^{4x}$

Applying leibnitz theorem

$$y^{(n)} = 4^n e^{4x} \cdot x^3 + n4^{(n-1)} e^{4x} \cdot 3x^2 + \frac{n(n-1)}{2!} 4^{(n-2)} e^{4x} \cdot 6x + \dots$$

$$\dots + \frac{n(n-1)(n-2)}{3!} 4^{(n-3)} e^{4x} \cdot 6$$

$$y^{(5)} = 4^5 e^{4x} x^3 + 5 \cdot 4^{(4)} e^{4x} \cdot 3x^2 + \frac{20}{2!} 4^3 e^{4x} \cdot 6x + \frac{60}{3!} 4^2 e^{4x} \cdot 6$$

$$y^{(5)} = 1024 x^3 e^{4x} + 3840 x^2 e^{4x} + 3840 x e^{4x} + 960 e^{4x}$$

$$y^{(5)} = 64 e^{4x} (16x^3 + 60x^2 + 60x + 15).$$

(ii) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ show that $x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$

solution

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

$$\equiv \underbrace{x^2 y^{(2)}}_{w_1} + \underbrace{x y^{(1)}}_{w_2} + \underbrace{y}_{w_3} = 0$$

w_1/

$U = y^2$ $\qquad V = x^2$

$U^n = y^{(n+2)}$ $\qquad V' = 2x$

$U^{(n-1)} = y^{(n+1)}$ $\qquad V^{(2)} = 2$

$U^{(n-2)} = y^{(n)}$

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w_2/

$u = y^{(1)}$ $\qquad$ $v = x$

$u^n = y^{(n+1)}$ $\qquad$ $v' = 1$

$u^{(n-1)} = y^n$

w_3/

$u = y$ $\qquad$ $v = 1$

$u^n = y^n$

$w_1 + w_2 + w_3 = 0$

Applying leibnits theorem

$$y^{(n+2)} \cdot x^2 + ny^{(n+1)} \cdot 2x + \frac{n(n-1)}{2!} y^n \cdot 2 + y^{(n+1)} \cdot x + ny^n + y^n = 0$$

$$x^2 y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^n + y^{(n+1)} \cdot x + ny^n + y^n = 0$$

$$x^2 y^{(n+2)} + 2xny^{(n+1)} + xy^{(n+1)} + n(n-1)y^n + ny^n + y^n = 0$$

$$x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n[n(n-1) + n + 1] = 0$$

$$x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n^2 \cancel{-n} \cancel{+n} + 1] = 0$$

$$x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n^2 + 1) = 0$$