EKERUCHE CHINWE
15/ENG02/020
COMPUTER ENGINEERING

1. If $y = e^{x^2+x}$

Show that

$y'' = y(2x+1) + 2y$

and hence give that

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

Soln

$y' = (2x+1)e^{x^2+x}$

$y'' = (2x+1)(2x+1)e^{x^2+x} + 2e^{x^2+x}$

$\underbrace{y''}_{w_1} = y'\underbrace{(2x+1)}_{w_2} + \underbrace{2y}_{w_3}$

for w_1

$y = n+2$

for ~~w~~ w_2

$u = y'$ $\quad$ $v = 2x+1$

$u^{(n)} = y^{(n+1)}$ $\quad$ $v' = 2$

$u^{(n-1)} = y^{(n)}$ $\quad$ $v'' = 0$

$y^{(n+1)}\, 2x+1 + n y^{(n)}\, 2$

for w_3

$2y^{(n)}$

$2y^n$

$w_1 = w_2 + w_3$

$y^{(n+2)} = y^{(n+1)}(2x+1) + ny^{(n)}2 + 2y^{n}$

$y^{(n+2)} = (2x+1)\,y^{(n+1)} + 2ny^{(n)} + 2y^{n}$

$y^{(n+2)} = (2x+1)\,y^{(n+1)} + 2(n+1)\,y^{n}$

2) $y = x^3 e^{4x}$, determine $y^{(5)}$

$u = e^{4x}$

$u^{n} = 4^{n} e^{4x}$ $\qquad v = x^3$

$u^{(n-1)} = 4^{n-1} e^{4x}$ $\qquad v^1 = 3x^2$

$u^{(n-2)} = 4^{n-2} e^{4x}$ $\qquad v^2 = 6x$

$u^{(n-3)} = 4^{n-3} e^{4x}$ $\qquad v^3 = 6$

$v^4 = 0$

$$y^{n} = u^{n}v + nu^{(n-1)}v^{1} + \frac{n(n-1)}{2!}u^{(n-2)}v^{2} + \frac{n(n-1)(n-2)\,u^{(n-3)}}{3!}v^{3}$$

$$+ \frac{n(n-1)(n-2)(n-3)\,u^{(n-4)}}{4!}v^{4}$$

$$y^{n} = 4^{n}e^{4x}x^{3} + n4^{n-1}e^{4x}3x^{2} + \frac{n(n-1)4^{n-2}}{2}e^{4x}6x +$$

$$\frac{n(n-1)(n-2)}{3!}4^{n-3}e^{4x}6$$

$y^{n} = 4^{n}e^{4x}x^{3} + n4^{n-1}e^{4x}3x^{2} + n(n-1)4^{n-2}e^{4x}3x$

$y^{5} = 4^{5}e^{4x}x^{3} + 5 \times 4^{4}e^{4x}3x^{2} + 5(5-1)4^{3}e^{4x}3x$

$y^{5} = 1024x^{3}e^{4x} + 3840x^{2}e^{4x} + 3840xe^{4x}$

2 $x^2 \frac{d^2y}{dx^2} + x\, dy/dx + y = 0$ show that $x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$

$\underbrace{x^2 y^2}_{w_1} + \underbrace{x y^{(1)}}_{w_2} + y = 0$

for w_1

$u = y^{(2)}$ $\quad v = x^2$

$u^{n(2)} = y^{(n+2)}$ $\quad v^{(1)} = 2x$

$u^{(n-1)} = y^{(n+1)}$ $\quad v^{(2)} = 2$

$u^{(n-2)} = y^{(n)}$ $\quad v^3 = 0$

$\therefore y^{(n+2)} x^2 + n y^{(n+1)} 2x + \frac{n(n-1)y^n 2}{2!} + 0$

for w_2 $\quad v = x$

$u = y^{(1)}$ $\quad v = 1$

$u^{(n)} = y^{(n+1)}$ $\quad v^2 = 0$

$u^{(n-1)} = y^{(n)}$

$= y^{(n+1)} x + n y^{(n)} \cdot 1 + 0$

for w_3

$y(n)$

$x^2 y^{(n+2)} + 2xn y^{(n+1)} + n(n-1)y^n + xy^{n+1} + ny^{(n)} + y^{(n)} = 0$

$x^2 y^{(n+2)} + 2nxy^{(n+1)} + n(n+1)y^n + xy^{n+1} + ny^{(n)} + y^n = 0$

$x^2 y^{n+2} + (2n+1)xy^{(n+1)} + [n(n-1)+n+1]y^n = 0$

$x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + [n^2-n+n+1]y^n = 0$

~~$x^2 y^n$~~ $x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + [n^2+1]y^n = 0$

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