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15/ENG04/016
Electrical & Electronic Engineering
ENG 381

(1) If $y = e^{x^2+x}$ show that

$$y'' = y'(2x+1) + 2y$$

and hence prove that

$$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$$

Soln

$$y' = (2x+1)e^{x^2+x}$$

$$y'' = (2x+1)(2x+1)e^{x^2+x} + 2e^{x^2+x}$$

$$\underbrace{y''}_{w_1} = \underbrace{y'(2x+1)}_{w_2} + \underbrace{2y}_{w_3}$$

For w1

$$y^{n+2}$$

For w2

$$u = y' \qquad v = 2x+1$$

$$u^n = y^{(n+1)} \qquad v' = 2$$

$$u^{(n-1)} = y^{(n)} \qquad v'' = 0$$

$$y^{(n+1)}2x+1 + ny^{(n)}2$$

For w3

$$2y^{(n)}$$

$$2y^n$$

$$w_1 = w_2 + w_3$$

$$y^{(n+2)} = y^{(n+1)}(2x+1) + ny^{(n)}2 + 2y^n$$

$$y^{(n+2)} = (2x+1)y^{(n+1)} + 2ny^{(n)} + 2y^n$$

$$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$$

(3) (i) $y = x^3 e^{4x}$, determine $y^{(5)}$

$u = e^{4x}$ & $v = x^3$

$u^n = 4^n e^{4x}$

$u^{(n-1)} = 4^{n-1} e^{4x}$

$u^{(n-2)} = 4^{n-2} e^{4x}$

$u^{(n-3)} = 4^{n-3} e^{4x}$

$v' = 3x^2$

$v^2 = 6x$

$v^3 = 6$

$v^4 = 0$

$$y^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v^2 + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v^3$$

$$+ \frac{n(n-1)(n-2)(n-3)}{4!} u^{(n-4)} v^4$$

$$y^n = 4^n e^{4x} x^3 + n 4^{n-1} e^{4x} 3x^2 + \frac{n(n-1)}{2} 4^{n-2} e^{4x^3} 6x +$$

$$\frac{n(n-1)(n-2)}{3!} 4^{n-3} e^{4x} 6$$

$$y^n = 4^n e^{4x} x^3 + n 4^{n-1} e^{4x} 3x^2 + n(n-1) 4^{n-2} e^{4x^3} 3x$$

$$y^5 = 4^5 e^{4x} x^3 + 5 \cancel{x} 4^4 e^{4x} 3x^2 + 5(5-1) 4^3 e^{4x^3} 3x$$

$$y^5 = 1024 x^3 e^{4x} + 3840 x^2 e^{4x} + 3840 x e^{4x^3}$$

20) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ Shew that $x^2y^{(n+2)} + (2n+1)xy^{(n+1)}$
$+ (n^2+1)y^{(n)} = 0$

$$\underbrace{x^2y^{(2)}}_{w_1} + \underbrace{xy^{(1)}}_{w_2} + \underbrace{y}_{w_3} = 0$$

For w_1

$u = y^{(2)}$ $\quad v = x^2$

$u^{(n)} = y^{(n+2)}$ $\quad v^{(1)} = 2x$

$u^{(n-1)} = y^{(n+1)}$ $\quad v^{(2)} = 2$

$u^{(n-2)} = y^{(n)}$ $\quad v^{(3)} = 0$

$$= y^{(n+2)}x^2 + ny^{(n+1)}2x + \frac{n(n-1)}{2!}y^n 2 + 0$$

For w_2

$u = y^{(1)}$ $\quad v = x$

$u^{(n)} = y^{(n+1)}$ $\quad v' = 1$

$u^{(n-1)} = y^{(n)}$ $\quad v^2 = 0$

$$= y^{(n+1)}x + ny^{(n)} \cdot 1 + 0$$

For $w3$

$y^{(n)}$

$$x^2y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^n + xy^{n+1} + ny^{(n)} + y^{(n)} = 0$$

$$x^2y^{(n+2)} + 2nxy^{(n+1)} + n(n+1)y^n + xy^{n+1} + ny^{(n)} + y^{(n)} = 0$$

$$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + [n(n-1) + n + 1]y^{(n)} = 0$$

$$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + [n^2 - n + n + 1]y^{(n)} = 0$$

$$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + [n^2 +]y^{(n)} = 0$$