IVWIGHREN OGHENE-OGAGA
15/ENG02/031
COMPT. ENG

ASSIGNMENT

1. If $y = e^{x^2+x}$

Show that $y'' = y'(2x+1) + 2y$

and hence prove that

$$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^{n}$$

SOLN

$$y' = (2x+1)e^{x^2+x}$$

$$y'' = (2x+1)(2x+1)e^{x^2+x} + 2e^{x^2+x}$$

$$\underbrace{y''}_{W_1} = \underbrace{y'(2x+1)}_{W_2} + \underbrace{2y}_{W_3}$$

for W1

$$y^{n+2}$$

for W2

$$u = y'$$

$$u^{(n)} = y^{(n+1)}$$

$$\cancel{\ } u^{(n-1)} = y^{(n)}$$

$$V = 2x+1$$

$$V' = 2$$

$$V'' = 0$$

$$y^{(n+1)} = 2x+1 + ny^{(n)}2$$

for W_3

$2y^{(n)}$

$2y^{n}$

$W_1 = W_2 + W_3$

$y^{(n+2)} = y^{(n+1)}(2x+1) + ny^{(n)}2 + 2y^{n}$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2ny^{(n)} + 2y^{n}$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^{n}$

2 $y = x^3 e^{4x}$ determine $y^{(5)}$

$u = e^{4x}$ $\quad$ $v = x^3$

$u^{n} = 4^{n}e^{4x}$ $\quad$ $v^1 = 3x^2$

$u^{(n-1)} = 4^{n-1}e^{4x}$ $\quad$ $v^2 = 6x$

$u^{(n-2)} = 4^{n-2}e^{4x}$ $\quad$ $v^3 = 6$

$u^{(n-3)} = 4^{n-3}e^{4x}$ $\quad$ $v^4 = 0$

$y^{n} = u^{n}v + nu^{(n-1)}v^{1} + \frac{n(n-1)}{2!}u^{(n-2)}v^{2} + \frac{n(n-1)(n-2)u^{(n-3)}}{3!}$

$v^{3} + \cancel{\frac{n(n-1)(n-2)(n-3)u^{(n-4)}}{4!}v^{4}} \to 0$

$y^{n} = 4^{n}e^{4x}x^{3} + n4^{n-1}e^{4x}3x^{2} + n(n-1)4^{n-2}e^{4x}\cdot 3x$

$y^{5} = 4^{5}e^{4x}x^{3} + 5\times 4^{4}e^{4x}5x^{2} + 5(5-1)4^{3}e^{4x}\cdot 3x$

$y^{5} = 1024x^{3}e^{4x} + 3840x^{2}e^{4x} + 3840xe^{4x}$

2ii) $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$ Show that $x^2y^{(n+2)}+(2n+1)xy^{(n+1)}+(n^2+1)y^{(n)}=0$

$$\underbrace{x^2y^{(2)}}_{W_1}+\underbrace{xy^{(1)}}_{W_2}+\underbrace{y}_{W_3}=0$$

for W_1

$u=y^{(2)}$ $\quad V=x^2$

$u^{(n)}=y^{(n+2)}$ $\quad V'=2x$

$u^{(n-1)}=y^{(n+1)}$ $\quad V^2=2$

$u^{(n-2)}=y^{(n)}$ $\quad V_3=0$

$$=y^{(n+2)}x^2+ny^{(n+1)}2x+\frac{n(n-1)}{2!}y^n 2+0$$

for W_2

$u=y^{(1)}$ $\quad V=x$

$u^{(n)}=y^{(n+1)}$ $\quad V=1$

$u^{(n-1)}=y^{(n)}$ $\quad V^2=0$

$$=y^{(n+1)}x+ny^{(n)}\cdot\underline{1}+0$$

for W_3

$y^{(n)}$

$$x^2y^{(n+2)}+2xny^{(n+1)}+n(n-1)y^n+xy^{n+1}+ny^{(n)}+y^{(n)}=0$$

$$x^2y^{(n+2)}+2nxy^{(n+1)}+n(n+1)y^n+xy^{n+1}+ny^{(n)}+y^{(n)}=0$$

$$x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + [n(n-1) + n + 1]y^{(n)} = 0$$
$$x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + [n^2 - n + n + 1]y^{(n)} = 0$$
$$x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + [n^2 + 1]y^{(n)} = 0$$