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15/ENG02/019
Computer Engineering
ENG 382 ASSIGNMENT 3

$y = e^{x^2+x}$ —— (i)

$y^n = a^n e^{ax}$

$y' = (2x+1)e^{x^2+x}$ —— (ii)

$u = 2x+1$ $\quad$ $v = e^{x^2+x}$

$\frac{du}{dx} = 2$ $\quad$ $\frac{dv}{dx}$ $(2x+1)e^{x^2+x}$

Using Product rule

$y'' = u\frac{dv}{dx} + v\frac{du}{dx}$

$y'' = (2x+1)(2x+1)e^{x^2+x} + 2e^{x^2+x}$

From eq 1 and (ii)

$y'' = y'(2x+1) + 2y$ - -

$y^{(2)} = y^{(1)}(2x+1) + 2y$

$W_1 = y^{(2)}$ $\quad$ $W_2 = y^{(1)}(2x+1)$ $\quad$ $W_3 = 2y$

$u = y^{(2)}$, $v = 1$ $\quad$ $u = y^{(1)}$ $\quad$ $v = 2x+1$ $\quad$ $u = y$ $\quad$ $v = 2$

$u^n = y^{(2+n)}$ $\quad$ $u^n = y^{(1+n)}$ $\quad$ $v' = 2$ $\quad$ $u^n = y^n$

$u^{n-1} = y^{(n)}$

$w_1^{(n)} = w_2^{(n)} + w_3^{(n)}$

$y^n = u^{(n)}v + nu^{(n-1)}v'$

$y^{(2+n)} = y^{(1+n)} \cdot (2x+1) + n(y^{(n)}).2 + y^{(n)}.2$

$y^{(2+n)} = (2x+1)y^{(1+n)} + 2(n+1)y^{(n)}$

2) $y = x^3 e^{4x}$ Find $y^{(5)}$

$u = e^{4x}$ $\quad$ $v = x^3$

$u^n = 4^n e^{4x}$ $\quad$ $v' = 3x^2$

$u^{n-1} = 4^{n-1}e^{4x}$ $\quad$ $v^2 = 6x$

$u^{n-2} = 4^{n-2}e^{4x}$ $\quad$ $v^3 = 6$

$u^{n-3} = 4^{n-3}e^{4x}$

$y^{(n)} = u^{(n)}v + nu^{(n-1)}v' + \frac{n(n-1)u^{(n-2)}}{2!}v^2 +$

$\frac{n(n-1)(n-2)}{3!}u^{(n-3)}v^{(3)} + \ldots$

2 $y^{(5)} = 4^5 e^{4x} \cdot x^3 + 5(4^4 e^{4x}) \cdot 3x^2 + \frac{5(4)(4^3 e^{4x})}{2!} \cdot 6x$

$+ \frac{5(4)(3)(4^2 e^{4x})}{3!} \cdot 6$

$y^{(5)} = 1024x^3 e^{4x} + 3840x^2 e^{4x} + 3840xe^{4x} + 960e^{4x}$

$y^{(5)} = 64e^{4x}[16x^3 + 60x^2 + 60x + 15)$

2b $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$w_1 = x^2 y^{(2)}$

$u = y^{(2)}$ $\quad V = x^2$

$u^{(n)} = y^{(2+n)}$ $\quad V' = 2x$

$u^{(n-1)} = y^{(1+n)}$ $\quad V^2 = 2$

$u^{(n-2)} = y^{(n)}$

$w_2 = xy^{(1)}$

$u = y^{(1)}$ $\quad V = x$

$u^{(n)} = y^{(1+n)}$ $\quad V' = 1$

$u^{(n-1)} = y^{(n)}$

$w_3 = y$

$u = y$ $\quad V = 1$

$u^n = y^{(n)}$

Using lebnitz theorem

$y^n = u^{(n)}v + nu^{(n-1)}V^{(1)} + \frac{n(n-1)u^{(n-2)}V^{(2)}}{2!}$

$w_1(n) = y^{(2+n)} \cdot x^2 + n(y^{(1+n)} \cdot 2x) + \frac{n(n-1)y^{(n)}}{2!} \cdot 2$

$w_1(n) = x^2 y^{(n+2)} + 2xny^{(1+n)} + n(n-1)y^{(n)}$

$w_2(n) = y^{(1+n)}x + ny^{(n)} \cdot 1$

$w_2^{(n)} = xy^{(1+n)} + ny^{(n)}$

$w_3^{(n)} = y^{(n)}.1 = y^{(n)}$

Adding together:

$x^2y^{(n+2)} + 2xny^{(1+n)} + n(n-1)y^{(n)} + xy^{(1+n)} + ny^{(n)} + y^{(n)} = 0$

$x^2y^{(n+2)} + xy^{(1+n)}[2n+1] + y^{(n)}(n(n-1)+n+1] = 0$

$x^2y^{(n+2)} + xy^{(1+n)}[2n+1] + y^{(n)}[n^2-n+n+1] = 0$

$x^2y^{(n+2)} + (2n+1)xy^{(1+n)} + (n^2+1)y^{(n)}$