ERHIANO AGHOGHO VICTOR
15/ENG01/005
CHEMICAL ENGR

1) If $y = e^{x^2+x}$

$u = x^2 + x$

$\frac{du}{dx} = 2x + 1$

$\frac{d}{} y = e^u$

$\frac{dy}{du} = e^u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= e^u \times 2x+1$

$2x+1\, e^u$ $\qquad u = x^2 + x$

$\frac{dy}{dx} = 2x+1\, e^{x^2+x}$

$\frac{d^2y}{dx^2} = 2e^{x^2+x} + (2x+1)(2x+1)\, e^{x^2+x}$

$= 2e^{x^2+x} + 4x^2+4x+1\, e^{x^2+x}$

$\frac{d^2y}{dx^2} = 2e^{x^2+x} + 4x^2+4x+1\, e^{x^2+x}$

$y'' = \frac{d^2y}{dx^2}$ $\qquad y' = \frac{dy}{dx}$ $\qquad y = e^{x^2+x}$

$y'' = y'(2x+1) + 2y$

$y'' = 2e^{x^2+x} + 4x^2+4x+1\, e^{x^2+x}$

$y'(2x+1) = (2x+1)(2x+1)e^{x^2+x}$

$= 4x^2+4x+1\, e^{x^2+x}$

$2y = 2e^{x^2+x}$

$y'(2x+1) + 2y = 2e^{x^2+x} + 4x^2+4x+1\, e^{x^2+x}$

$= 2e^{x^2+x} + 4x^2+4x+1\, e^{x^2+x}$

y''

$y'' = y'(2x+1) + 2y$

$w_1 \qquad w_2 \qquad w_3$

w_1

$u = y'' \qquad v = 1$

$u^n = y^{n+2} \qquad v = 0$

$= y^{n+2}.1 + 0$

w_2

$u = y' \qquad v = 2x+1$

$u^n = y^{n+1} \qquad v' = 2$

$u^{n-1} = y^n \qquad v = 0$

$= y^{n+1}(2x+1) + n(y^n).2 + 0$

$= y^{n+1}(2x+1) + 2n(y^n)$

w_3

$u = \cancel{2}y \qquad v = 1$

$u^n = y^n \qquad v' = 0$

$= 2[(y^n.1) + 0]$

$= 2y^n$

$w_1 = w_2 + w_3$

$y^{n+2} = y^{n+1}(2x+1) + 2n(y^n) + 2y^n$

$= y^{n+1}(2x+1) + 2(n+1)y^n$

2a) Using the Leibnitz theorem given that $y = x^3e^{4x}$ determine $y^{(5)}$

Solutoon

$u = e^{4x} \qquad v = x^3$

$u^{(5)}v + 5u^4v' + 10u^3v'' + 10u^2v^3 + 5u'v^4 + uv^5$

$10(4^3e^{4x}6x) + 5(4^2e^{4x}.6) + 0$

$= 1024e^{4x}x^3 + 1280e^{4x}.3x^2 + 640e^{4x}.6x + 80e^{4x}.6$

$= 1024e^{4x}x^3 + 3840e^{4}x^2 + 3840e^{4x}x + 480e^{4x}$

u) $x^2\frac{dy^2}{dx^2} + x\frac{dy}{dx} + y = 0$

$x^2y'' + xy' + y = 0$

$\downarrow \quad \downarrow \quad \downarrow$

$w_1 \quad w_2 \quad w_3$

$w_1 + w_2 + w_3 = 0$

for w_1

$u = y'' \qquad v = x^2$

$u^n = y^{n+2} \qquad v' = 2x$

$u^{n-1} = y^{n+1} \qquad v'' = 2$

$u^{n-2} = y^n \qquad v''' = 0$

$= y^{(n+2)}(x^2) + n(y^{n+1})2x + \frac{n(n-1)y^n.2}{2!} + 0$

$x^2y^{(n+2)} + 2nx(y^{n+1}) + n(n-1)y^n$

for w_2

$u = y' \qquad u = x$

$u^n = y^{n+1} \qquad v' = 1$

$u^{n-1} = y^n \qquad v'' = 0$

$= y^{n+1}.x + ny^n + 0$

for w_3

$u = y \qquad v = 1$

$u^n = y^n \qquad v' = 0$

$= y^n.1$

$w_1 + w_2 + w_3 = 0$

$x^2y^{n+2} + 2nxy^{n+1} + (n^2-n)y^n + xy^{n+1} + ny^n + y^n$

$x^2y^{n+2} + 2nxy^{n+1} + xy^{n+1} + n^2y^n - \cancel{ny^n} + \cancel{ny^n} + y^n$

$x^2y^{n+2} + 2n+1(xy^{n+1}) + (n^2+1)y^n$