Rotimi Esther Aramide
15/ENG04/053

$$y = e^{x^2+x}$$

① (i) $y = (2x+1)e^{x^2+x}$

$y'' = (2x+1)(2x+1)(e^{x^2+x}) + 2(e^{x^2+x})$

$y'' = (2x+1)(2x+1)(e^{x^2+x}) + 2(e^{x^2+x})$

and $y = 1e^{x^2+x}$, $y = (2x+1)e^{x^2+x}$

$\therefore y'' = 2x+1(y^{(1)}) + 2(y^{(0)})$

(ii) $y'' = 2x+1(y^{(1)}) + 2(y^{(0)})$

$y'' - 2x+1(y^{(1)}) - 2(y^{(0)}) = 0$

$\underbrace{y^{(2)}}_{w_1} - \underbrace{2x+1(y^{(1)})}_{w_2} - \underbrace{2(y^{(0)})}_{w_3} = 0$

$w_1^{(n)} = y^{(n+2)}$

$w_2^{(n)} = y^{(n+1)}(2x+1) + ny^n(2)$

$w_3 = y^{(n)}$

$\therefore y^{(n+2)} - \{y^{(n+1)}(2x+1) + ny^n(2)\} - 2y^{(n)}$

$0 = y^{(n+2)} - y^{(n+1)}(2x+1) - 2ny^{(n)} - 2y^{(n)}$

$0 = y^{(n+2)} - y^{(n+1)}(2x+1) - \{2ny^{(n)} + 2y^{(n)}\}$

$0 = y^{(n+2)} - y^{(n+1)}(2x+1) - 2y^{(n)}\{n+1\}$

$y^{(n+2)} = y^{(n+1)}(2x+1) + 2y^{(n)}\{n+1\}$

$\therefore y^{(n+2)} = 2x+1(y^{(n+1)}) + 2(n+1)\{y^{(n)}\}$

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Rahmat Esther Aramide
15/ENG01/053

(2)(a) ii $y = x^3 e^{4x}$

let $u = e^{4x}$

$v = x^3$

From Leibnitz theorem

$$y^{(n)} = u^{(n)}v + nu^{(n-1)}v' + \frac{n(n-1)}{2!}u^{(n-2)}v'' + \cdots$$

$$\cdots + \frac{n(n-1)(n-2)u^{n-3}v^3}{3!} \cdots$$

$u = e^{4x}$	$v = x^3$
$u^{(1)} = 4e^{4x}$	$v' = 3x^2$
$u^{(2)} = 16e^{4x}$	$v'' = 6x$
$u^{(3)} = 64e^{4x}$	$v^3 = 6$
$u^{(4)} = 256e^{4x}$	$v^4 = 0$
$u^{(5)} = 1024e^{4x}$	$v^5 = 0$

$$y^{(5)} = 1024e^{4x}x^3 + 5 \cdot 256e^{4x}3x^2 + \cdots$$

$$\cdots + \frac{5(4)64e^{4x}6x}{2!} + \frac{5(4)(3)16e^{4x}6}{3!} + \cdots$$

$$\cdots + \frac{5(4)(3)(2)4e^{4x}(0)}{4!} + \frac{5(4)(3)(2)(1)e^{4x}0}{5!}$$

$$y^{(5)} = 1024e^{4x}x^3 + 3840e^{4x}x^2 + 3840e^{4x}x + 960e^{4x}.$$

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(11) $$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

$$x^2 y^{(2)} + x y^{(1)} + y^{(0)} = 0$$

$w_1 \qquad w_2 \qquad w_3$

$$w_1^{(n)} y^n = x^2 y^{(n+2)} + n y^{(n+1)} 2x + \frac{n(n-1) y^{(n)} \cdot 2}{2!} + 0$$

$$w_2^{(n)} y^n = x(y^{(n+1)}) + n y^{(n)} + 0$$

$$w_3^{(n)} = y^{(n)}$$

$$x^2 y^{(n+2)} + n y^{(n+1)} 2x + \frac{n(n-1) y^{(n)} \cdot 2}{2!} + x(y^{(n+1)}) + n y^{(n)} + \cancel{y^{(n)}}$$

$$+ y^{(n)} = 0$$

$$x^2 y^{(n+2)} + y^{(n+1)} 2xn + n(n-1) y^{(n)} + x(y^{(n+1)}) + n y^{(n)} + y^{(n)} = 0$$

$$x^2 y^{(n+2)} + y^{(n+1)} [2xn + x] + y^{(n)} [n(n-1) + n + 1] = 0$$

$$x^2 y^{(n+2)} + y^{(n+1)} [x(2n+1)] + y^{(n)} [n^2 - \cancel{n} + \cancel{n} + 1] = 0$$

$$x^2 y^{(n+2)} + y^{(n+1)} x [2n+1] + y^{(n)} [n^2+1] = 0$$

$$\therefore \; x^2 y^{(n+2)} + x(2n+1) y^{(n+1)} + y^{(n)} [n^2+1] = 0$$