If $y = e^{x^2+x}$

show that

$y'' = y'(2x+1) + 2y$

$y'' - y'(2x+1) - 2y = 0$

$u = y''$ $\quad v = 1$

$u = y^{(2)}$

$u^n = y^{(n+2)}$

$u^{(n-1)} = y^{(n+1)}$

$u^{(n-2)} = y^{(n)}$

$y^{(n+2)}$

$u = y'(2x+1)$

$u = y^{(1)}$ $\quad v = 2x+1$

$u^n = y^{(1+n)}$ $\quad v' = 2$

$u^{(n-1)} = y^{(n)}$

$y^{(n+1)}(2x+1) + ny^{(n)} \cdot 2$

$u = 2y$ $\quad v = 2$

$u = y^{(\cdot)}$

$u^n = y^{(n)}$

$2y^{(n)}$

$y^{(n+2)} - \left(y^{(n+1)}(2x+1) + 2ny^{(n)} + 2y^{(n)}\right) = 0$

$y^{(n+2)} = y^{(n+1)}(2x+1) + 2y^{(n)}(n+1)$

Integrated Data Services Limited
36, Ogba Road, Oko, P.M.B. 1003, Benin City, Edo State, Nigeria.

$y = e^{x^2+x}$

$y^n = a^n e^{ax}$

$\cancel{y^{n} = a^2 e^{x^2+x}}$

$y' = (2x+1)e^{x^2+x}$

$u = 2x+1 \qquad v = e^{x^2+x}$

$u' = 2 \qquad v' = (2x+1)e^{x^2+x}$

$y'' = (2x+1)(2x+1)e^{x^2+x} + 2(e^{x^2+x})$

$y'' = (2x+1)y' + 2y$

(ii) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$x^2y'' + xy' + y = 0$

$u = x^2y''$

$u = y^{(2)} \qquad v = x^2$

$u^{(n)} = y^{(n+2)} \qquad v' = 2x$

$u^{(n-1)} = y^{(n+1)} \qquad v'' = 2$

$u^{(n-2)} = y^{(n)} \qquad v''' = 0$

$y^{(n+2)}x^2 + n\,y^{(n+1)}\cdot 2x$

$+ \frac{n(n-1)}{2} y^{(n)}\cdot 2$

$u = y^{(1)} \qquad v = x$

$u^n = y^{(n+1)} \qquad v' = 1$

$u^{(n-1)} = y^{(n)} \qquad v'' = 0$

$y^{(n+1)}\cdot x + n\,y^{(n)}$

$u = y^{(1)} \qquad v = 1$

$u^n = y^n$

$= y^{(n)}$

$x^2y^{(n+2)} + 2n\,y^{(n+1)} + n(n-1)y^{(n)} + x\,y^{(n+1)}$
$+ n\,y^{(n)} + y^{(n)}$

$x^2y^{(n+2)} + y^{(n+1)}x(2n+1) + y^{(n)}(n(n-1) + n +$

$x^2y^{(n+2)} + (2n+1)x\,y^{(n+1)} + (n^2 \cancel{-n} \cancel{+n} + 1)y^{(n)}$

$= x^2y^{(n+2)} + (2n+1)x\,y^{(n+1)} + (n^2+1)y^{(n)}.$

IDSL... Centre of Technological Excellence

$y'' = y'(2x+1) + 2y$

$y = e^{x^2+x}$

$y^5 = 1024e^{4x} + 3840e^{4x}x^2 + 3840e^{4x}x + 960e^{4x}$

2) $y = x^3 e^{4x}$ determine $y^{(5)}$

$u^n = 4^n e^{4x}$ $\quad u^{n-3} = 4^{(n-3)}$

$u^{n-1} = 4^{(n-1)} e^{4x}$

$u^{n-2} = 4^{(n-2)} e^{4x}$

$u = e^{4x}$, $u^{(5)} = 4^5 e^{4x}$

$v = x^3$, $v' = 3x^2$, $v'' = 6x$, $v''' = 6$, $v^{(4)} = 0$

using lebniths theorem

$y^{(5)} = u^{(5)}v + 5u^4v' + u^3v^2 + 5u^2v^3 +$

$y^{(n)} = u^{(n)}e^{4x} + n\,4^{(n-1)}e^{4x}3x^2 + \frac{n(n-1)}{2}4^{(n-2)}e^{4x}6x + \frac{n(n-1)(n-2)}{3!}4^{(n-3)}e^{4x}$

$y^{(5)} = u^5e^{4x} + 5\cdot 4^4e^{4x}3x^2 + \frac{5(5-1)}{2}4^{(5-2)}e^{4x}6x + \frac{5(5-1)(5-2)}{3!}\cdot 4^{(5-3)}e^{4x}\cdot 6$

$y^5 = 4^5e^{4x} + 3840e^{4x}x^2 + \frac{7680}{2}e^{4x}x + 960e^{4x}$

IDSL... Centre of Technological Excellence