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MATRIC: 15/ENG02/032
COURSE: ENG 381 (Assignment)

2) Using the Lebnitz theorem given that

(i) $y = x^3 e^{4x}$ determine $y^{(5)}$

Solution

Recall that Lebnitzs theorem States that

$y^5 = U^5V + 5U^4V' + 10U^3V^2 + 10U^2V^3 + 5U'V^4 + V^5$

where $U = e^{4x}$ $\quad V = x^3$

$U' = 4e^{4x}$ $\quad V' = 3x^2$

$U^2 = 16e^{4x}$ $\quad V^2 = 6x$

$U^3 = 64e^{4x}$ $\quad V^3 = 6$

$U^4 = 256e^{4x}$

$U^5 = 1024e^{4x}$

therefore

$y^5 = 1024e^{4x}(x^3) + 5(256e^{4x})(3x^2) + 10(64e^{4x})(6x) + 10(16e^{4x})$
$\times 6 + 5(4e^{4x})(0) + 0^5$

$= 1024x^3 \cdot e^{4x} + 3840x^2e^{4x} + 3840xe^{4x} + 960e^{4x}$

$y^5 = e^{4x}(1024x^3 + 3840x^2 + 3840x + 960)$

(ii) If $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$; ~~Show that~~ $\cancel{x^2 y^{(n+2)}}$ +

Show that $x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$

Solution

The equation can be written as $x^2y'' + xy' + y = 0$

Let $w_1 = x^2y''$, $w_2 = xy'$ and $w_3 = y$

solving for $w_1 = x^2y''$

let $U = y^2$: $U^n = y^{n+2}$

let $V = x^2$; $V' = 2x$ $\quad V'' = 2$; $V''' = 0$

$w_1 = y^{(n+2)}x^2 + ny^{(n+1)}2x + \frac{n(n-1)}{2!}y^n \times 2 + 0$

$W_2 = xy'$

let $u = y'$ $u^n = y^n$

let $v = x$ $v' = 1$ and $v'' = 0$

$W_2 = y^{(n+1)} \cdot x + ny^n * 1 + 0$

$= xy^{n+1} + ny^n$

$W_3 = y^{(n)}$

Combining

$W = W_1 + W_2 + W_3$

$W = x^2 y^{(n+2)} + ny^{(n+1)} \cdot 2x + \frac{n(n-1)}{2} y^n \times 2 + xy' + y^{(n+1)}x$
$+ ny^n + y^n = 0$

$x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n[n(n-1) + n + 1] = 0$

$x^2 y^{(n+2)} + (2n+1)xy^{n+1} + (n^2+1)y^n = 0$

(1) If $y = e^{x^2+x}$ Show that $y'' = y'(2x+1) + 2y$ and hence prove that $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

Solution

$y = e^{x^2+x}$

let $u = x^2 + x$

$\frac{du}{dx} = 2x + 1$

$y = e^u$

$\frac{dy}{du} = e^u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$\frac{dy}{dx} = e^u \times 2x+1$

where $u = x^2 + x$

$\frac{dy}{dx} = e^{x^2+x} \cdot (2x+1) = y'$

If $y' = e^{x^2+x}(2x+1)$

then $y'' = 2e^{x^2} \cdot e^{x^2+x}(2) + \left[e^{x^2+x} \cdot (2x+1)\right](2x+1)$

$y'' = 2 \cdot e^{x^2+x} + e^{x^2+x} \cdot (2x+1) \cdot (2x+1)$

but $y = e^{x^2+x}$

and $y' = e^{x^2+x}(2x+1)$

$y'' = 2(y) + y'(2x+1)$

Applying Leibnitz theorem to the above equation finding the nth derivative

$$y^{(n+2)} = 2(n+1)y^{n} + y^{(n+1)}(2x+1)$$