Alamin Muhammed Nabil .O
15/ENG02/006
Computer Engineering

1) $y = e^{x^2+x}$; show that: $y'' = y'(2x+1) + 2y$

Soln

$y'' = y'(2x+1) + 2y$ ——— $(*)$

~~solving LHS~~

$y'' = \frac{d^2y}{dx^2}$ $y' = \frac{dy}{dx}$

$\therefore \frac{dy}{dx} = (2x+1)e^{x^2+x}$

Using product rule $\Rightarrow V\frac{du}{dx} + U\frac{dv}{dx}$

$\frac{d^2y}{dx^2} = (2x+1)(2x+1)e^{x^2+x} + 2e^{x^2+x}$

$= (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$

$= (4x^2+4x+1)e^{x^2+x} + 2e^{x^2+x}$

$= (4x^2+4x+1+2)e^{x^2+x}$

$= (4x^2+4x+3)e^{x^2+2x}$

Substituting RHS & LHS g above ~~stated~~ eqn $*$

$(4x^2+4x+3)e^{x^2+x} = (2x+1)e^{x^2+x}(2x+1) + 2(e^{x^2+x})$

$= (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$

$= (4x^2+4x+1+3)e^{x^2+x}$

$= (4x^2+4x+4)e^{x^2+x}$

$(4x^2+4x+3)e^{x^2+x} = (4x^2+4x+3)e^{x^2+x}$

$\therefore y'' = y'(2x+1) + 2y$

//

1.i) $y'' = y'(2x+1) + 2y$

$y'' - y'(2x+1) - 2y = 0$

let $W = y''$

$V = 1 \quad V' = 0$

$U = y'' \quad U^n = y^{n+2}$

$W^n = U^n V + n U^{n-1} V'$

$= y^{(n+2)} + 0$

let $W = -y'(2x+1)$

$V = 2x+1 \quad V' = 2 \quad V'' = 0$

$U = -y' \quad U^n = -y^{n+1}$

$W^n = U^n V + n U^{n-1} V' + \frac{n(n-1)}{2!} U^{n-2} V''$

$= -y^{n+1}(2x+1) + n(-y^{n+1-1})(2) + 0$

$= -y^{n+1}(2x+1) + 2n(-y^n)$

let $W = -2y$

$V = -2 \quad V' = 0$

$U = y \quad U^n = y^n$

$W^n = U^n V + n U^{n-1} V'$

$= y^n - 2 + 0$

$= -2y^n$

$\therefore y^n = y^{n+2} - y^{n+1}(2x+1) + 2n(-y^n) - 2y^n$

$y^{n+2} - y^{n+1}(2x+1) + 2n(-y^n) - 2y^n = 0$

$y^{n+2} - y^{n+1}(2x+1) - 2y^n(n+1) = 0$

$y^{(n+2)} = y^{(n+1)}(2x+1) + 2y^n(n-1)$

2i) $y = x^3 e^{4x}$ find y^5

$V = x^3$, $V' = 3x^2$, $V'' = 6x$, $V''' = 6$, $V^4 = 0$
$u = e^{4x}$, $u' = 4e^{4x}$, $u^2 = 16e^{4x}$, $u^3 = 64e^{4x}$, $u^4 = 256e^{4x}$ +
$u^5 = 1024e^{4x}$.

$$y^n = u^n V + nV^{n-1}V' + \frac{n(n-1)}{2!}u^{n-2}V^2 + \frac{n(n-1)(n-2)}{3!}u^{n-3}V^3 + \frac{n(n-1)(n-2)(n-3)}{4!}u^{n-4}V^4$$

$$y^5 = u^5 V + 5u^4 V' + \frac{5(4)}{2\times1}u^3V^2 + \frac{5(4)(3)}{3\times2\times1}u^2V^3 + 0$$

$$y^5 = 1024e^{4x}(x^3) + 15x^2(256e^{4x}) + 60x(64e^{4x}) + 60(16e^{4x})$$

$$y^5 = x^3 1024e^{4x} + x^2 3840e^{4x} + x\,3840e^{4x} + 960e^{4x}$$

$$y^5 = 64e^{4x}(16x^3 + 60x^2 + 60x + 15)$$

2iii) $x^2 \frac{dy^2}{dx} + x\frac{dy}{dx} + y = 0$

$\Rightarrow \quad x^2y'' + xy' + y = 0$

let $W = x^2y''$

$V = x^2, \quad V' = 2x \quad V'' = 2 \quad V''' = 0$

$U = y'' \quad U^n = y^{n+2}$

$W^n = U^nV + nU^{n-1}V' + \frac{n(n-1)}{2!}U^{n-2}V'' + \frac{n(n-1)(n-2)}{3!}U^{n-2}V'''$

$= y^{n+2}(x^2) + n(y^{n+2-1})(2x) + \frac{n(n-1)}{2!}(y^{n+2-2})(2)$

$= x^2y^{n+2} + 2xn(y^{n+1}) + n(n-1)(y^n)$

let $W = xy'$

$V = x \quad V' = 1 \quad V'' = 0$

$U = y' \quad U^n = y^{n+1}$

$W^n = y^{n+1}(x) + n(y^{n+1-1})(1) + 0$

$= xy^{n+1} + ny^n$

let $W = y$

$V = 1 \quad V' = 0$

$U = y \quad U^n = y^n$

$W^n = y^n$

$y^n = x^2y^{(n+2)} + 2xn(y^{n+1}) + n(n-1)(y^n) + xy^{n+1} + ny^n + y$

$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2-n+n+1)y^n = 0$

$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$ //

Scanned by CamScanner