NDIFE NZUBE KINGSLEY

15/ENG05/014

MECHATRONICS

① $y = e^{x^2+x}$

$\frac{dy}{dx} = (2x+1) e^{x^2+x}$

$= 2e^{x^2+2} + (2x+1)(2x+1) e^{x^2+x}$

$\Rightarrow e^{x^2+x} [(12x+1)(2x+1) + 2)$

$y' = (2x+1) + 2y$

$(2x+1) e^{x^2+x} \quad (2x+1) + 2y$

$(2x+1) e^{x^2+x} (2x+1) + 2e^{x^2+x}$

$e^{x^2+x} [2x+1)(2x+1) + 2)$

$y'' = \underset{w_1}{y'} \underset{w_2}{(2x+1)} + \underset{w_3}{2y}$

$w_1 = u = y^2$ $\quad$ $w_2 \; u = y'$ $\quad$ $v = 2x+1$

$u^n = y^{(n+2)}$ $\quad$ $u^n = y^{n+1}$ $\quad$ $v' = 2$

$u = y$ $\quad$ $v = 2$

$u^n = y^n$ $\quad$ $v = 0$

$w_1 = w_2 + w_2$

$y^{n+2} = y^{n+1}(2x+1) + ny^n \cdot 2 + y^n 2$

$= y^{(n+1)}(2x+1) + 2ny^n + 2y^n$

$= [2x+1] y^{(n+1)} + 2[n+1] y^n$

NDIFE NZUBE KINGSLEY
15/ENG05/014
MECHATRONICS

(2b) (2)

$y = x^3 e^{4x}$

Solution

Let $v = x^3$ $\qquad u = e^{4x}$

$v^0 = x^3$ $\qquad u^0 = e^{4x}$

$v' = 3x^2$ $\qquad u^n = 4^n e^{4x}$

$v'' = 6x$ $\qquad u^{n-1} = 4^{n-1} e^{4x}$

$v''' = 6$ $\qquad u^{n-2} = 4^{n-2} e^{4x}$

$v^{iv} = 0$ $\qquad u^{n-3} = 4^{n-3} e^{4x}$

$\qquad u^{n-4} = 4^{n-4} e^{4x}$

$$u^n v^0 + \frac{n u^{n-1} v^1}{1} + \frac{n(n-1) u^{n-2} v^2}{1\cdot 2} + \frac{n(n-1)(n-2) u^{n-3} v^3}{1\cdot 2\cdot 3}$$

$$4^n \cdot x^3 + n 4^{n-1} e^{4x} \cdot 3x^2 + \frac{n(n-1) 4^{n-2}}{1\cdot 2} \cdot 6x + \frac{n(n-1)(n-2) 4^{n-3}}{1\cdot 2\cdot 3} \cdot 6$$

$n = 5$

$$4^{(5)} \cdot x^3 + 5(4^{5-1}) \cdot e^{4x} \cdot 3x^2 + \frac{5(5-1) 4^{5-2}}{1\cdot 2} \cdot 6x + \frac{5(5-1)(5-2) 4^{5-3}}{1\cdot 2\cdot 3} \cdot 6$$

$= \cancel{\ldots}\, x^3 \cancel{\ldots} +$

$$= 4^5 x^3 + 5(4^4) e^{4x} \cdot 3x^2 + 10(4)^3 \cdot 6x + 5(4)(3)4^2 \cdot 6$$

Scanned by CamScanner

NDIFE NZUBE KINGSLEY

15/ENG05/014

MECHATRONICS ENGINEERING

(2) using leibnitz

(i) $y = x^3 e^{4x}$

Solution:

from: $e^{ax} = a^n e^{ax}$

$y = 4^n x^3 e^{4x}$

$y^{(5)} = 4^{(5)} x^3 e^{4x} = 1024 x^3 e^{4x}$

1280 ~~12~~

(2 ii)

(ii) $x^2 \frac{d^2y}{dx} + x\frac{dy}{dx} + y = 0$

show $x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^n = 0$

Solution

Let $w^n = x^2 y''$

Let $v = x^2$, $u = y''$

$v^0 = x^2$ $u^0 = y''$

$v' = 2x$ $u' = y'''$

$v'' = 2$ $u'' = y^{iv}$

$v''' = 0$ $u''' = y^{v}$ $u^n = y^{n+2}$

Scanned by CamScanner

From : Leibnitz :

$$U^n V^0 + \frac{n U^{n-1}}{1} V' + \frac{n(n-1) U^{n-2}}{1 \cdot 2} U^2 + \frac{n(n-1)(n-2) U^{n-3}}{1 \cdot 2 \cdot 3} U^3 + \dots$$

$$\Rightarrow y^{n+2} \cdot x^2 + n y^{n+1} \cdot 2x + \frac{n(n-1) y^n}{1 \cdot 2} \cdot 2 + \cancel{\frac{n(n-1)(n-2) y^{n-1}}{1 \cdot 2 \cdot 3}} \cdot 0 \xrightarrow{} 0$$

$$w^n \Rightarrow x^2 y^{n+2} + 2xn y^{n+1} + n(n-1) y^n$$

Again; let $w_2^n = y'$

$V = x, \quad u = y'$

$V^0 = x \quad u^0 = y'$

$V' = 1 \quad u' = y''$

$V'' = 0 \quad y^n = y^{n+1}$

using the above equation (b)

$$U^n V^0 + \frac{n U^{n-1}}{1} V' + \cancel{\frac{n(n-1) U^{n-2}}{1 \cdot 2}} U^2 \xrightarrow{} 0$$

$$y^{n+1} \cdot x + \frac{n y^n}{1} \cdot 1 +$$

$$w^n \Rightarrow y^{n+1}$$

(b) $\omega_3^n = y^n$

$\Rightarrow$ $x^2 y^{n+2} + 2xny^{n+1} + n(n-1)y^n + xy^{n+1} + y^n = 0$

$x^2 y^{n+2} + 2xny^{n+1} + (n^2 - n)y^n + xy^{n+1} + y^n = 0$

$x^2 y^{n+2} + xy^{n+1}[2n+1] + (n^2-n)y^n = 0$

If $y = e^{x^2+x}$

show $y = y(2x+1) + 2y$

prove that $y^{n+2} = (2x+1)y^{n+1} + 2(n+1)y^n$