If $x^2y'' + xy' + y = 0$, show that $x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y = 0$

soln $\cdot\ \omega = x^2 y''$

$V = x^2 \quad = dv/dU = 2x \ ,\ d/dv^2 = 2 \ ,\ d/dv^3 = 0$

$U = y'' \qquad U^n = y^{(n+2)}$

$W^{(n)} = U^{(n)}V + nU^{(n-1)}V' + \frac{n(n-1)}{2!}U^{(n-2)}V^2 + \frac{n(n-1)(n-2)}{2+3!}U^{(n-3)}V^3$

$= y^{(n+2)}x^2 + ny^{n+2-1} \cdot 2x + \frac{n(n-1)}{2} \cdot y^{n+2}$

$\Rightarrow y^{(n+2)}x^2 + 2n\,xy^{(n+1)} + n(n-1)y^{(n)}$

$W = xy' ,\ V = x \ ,\ V' = 1 ,\ V^2 = 0$

$U = y' \ ,\ U^n = y^{(n+1)}$

$\cdot\ W^{(n)} = y^{(n+1)}x + ny^{n+1-1} + 0$

$= y^{(n+1)}x + ny^n$

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$u = y$, $V = 1$, $V' = 0$, $u = y$ $u^n = y^{(n)}$

$K^{(n)} = y^n$

Then $[x^2 y'' + xy' + y]^n = 0$ becomes

$y^n = x^2 y^{(n+2)} + 2xny^{(n+1)} + x y^{(n+1)} + n(n-1) y^n + ny^n + y^n$

$y^n = x^2 y^{(n+2)} + (2n+1) xy^{(n+1)} + (n^2 - n) y^{(n)} + y^{(n)}$

$y^n = x^2 y^{(n+2)} + (2n+1) xy^{(n+1)} + n^2 + 1) y^{(n)} = 0$