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15/ENG01/003

CHEMICAL ENGINEERING

Assignment 3

1 If $y = e^{x^2+x}$

$u = x^2 + x$

$\frac{du}{dx} = 2x + 1$

$y = e^u$

$\frac{dy}{du} = \cancel{e^u}\ e^u$

$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du}$

$= \cancel{2x}+1 \times e^u$

$dy/dx = 2x + e^{x^2+x}$

$d^2y/dx^2 = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$= 2e^{x^2+x} + 4x^2+4x+1e^{x^2+x}$

$\frac{d^2y}{dx^2} = 2e^{x^2+x} + 4x^2+4x+e^{x^2+x}$

$y'' = y'(2x+1) + 2y$

$y'' = 2e^{x^2+x} + 4x^2+4x+1e^{x^2+x}$

$y'(2x+1) = (2x+1)(2x+1)e^{x^2+x}$

$= 4x^2+4x+1e^{x^2+x}$

$2y = 2e^{x^2+x}$

$y'(2x+1) + 2y = 2e^{x^2+x} + 4x^2+4x+1e^{x^2+x}$

$= 2e^{x^2+x} + 4x^2+4x+1e^{x^2+x}$

let $y'' = P_1$

$y'(2x+1) = P_2$

$2y = P_3$

P_1

$u = y''$ $\quad v = 1$

$u^n = y^{n+2}$ $\quad v = 0$

$= y^{n+2} \cdot 1 + 0$

P_2

$u = y'$ $\quad v = 2x+1$

$u^n = y^{n+1}$ $\quad v' = 2$

$u^{n-1} = y^n$ $\quad v'' = 0$

$= y^{n+1}(2x+1) + n(y^n) \cdot 2 + 0$

$= y^{n+1}(2x+1) + 2n(y^n)$

P_3

$u = 2y$ $\quad v = 1$

$y^n = y^n$ $\quad v' = 0$

$= 2[(y^n \cdot 1) + 0]$

$= 2y^n$

$P_1 = P_2 + P_3$

$y^{n+2} = y^{n+1}(2x+1) + 2n(y^n) + 2y^n$

$= y^{n+1}(2x+1) + 2(n+1)y^n$

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2a. From leibnitz theorem
if $y = x^3 e^{4x}$

$u = e^{4x}$ $\quad v = x^3$

By

$y^{(5)} = u^{(5)} v + 5u^4 v' + 10u^3 v'' + 10u^2 v^3 + 5u' v^4 + u v^5$

$= 4^5 e^{4x} \cdot x^3 + 5(4^4 e^{4x} \cdot 3x^2) + 10(4^3 e^{4x} 6x) + 5(4^2 e^{4x} 6) + 0$

$= 1024 e^{4x} x^3 + 1280 e^{4x} \cdot 3x^2 + 640 e^{4x} \cdot 6x + 80 e^{4x} \cdot 6$

$= 1024 e^{4x} x^3 + 3840 e^{4x} x^2 + 3840 e^{4x} x + 480 e^{4x}$

b. $x^2 \frac{dy^2}{dx^2} + x \frac{dy}{dx} + y = 0$

Let $w_1 = x^2 y''$

$w_2 = x y'$

$w_3 = y$

$w_1 + w_2 + w_3 = 0$

For w_1

$u = y''$ $\quad v = x^2$

$u^n = y^{n+2}$ $\quad v' = 2x$

$u^{n-1} = y^{n+1}$ $\quad v'' = 2$

$u^{n-2} = y^n$ $\quad v''' = 0$

$\Rightarrow y^{(n+2)}(x^2) + n(y^{n+1}) 2x + \frac{n(n-1) y^n \cdot 2}{2 \times 1} + 0$

$= x^2 y^{n+2} + 2nx(y^{n+1}) + n(n-1) y^n$

For w_2

$u = y'$ $\quad v = x$

$u^n = y^{n+1}$ $\quad v' = 1$

$u^{n-1} = y^n$ $\quad v'' = 0$

$\Rightarrow y^{n+1} \cdot x + n y^n + 0$

for w_3

$u = y$ $\qquad v = 1$

$u^n = y^n$ $\qquad v' = 0$

$u^{n-1} = y^{n-1}$

$= y^n \cdot 1$

since $w_1 + w_2 + w_3 = 0$

$x^2 y^{n+2} + 2nx y^{n+1} + (n^2 - n) y^n + x y^{n+1} + n y^n + y^n$

$x^2 y^{n+2} + 2nx y^{n+1} + x y^{n+1} + n^2 y^n \cancel{- n y^n} \cancel{+ n y^n} + y^n$

$x^2 y^{n+2} + 2n+1 (x y^{n+1}) + (n^2 + 1) y^n$