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1) If $y = e^{x^2+x}$

$u = e^{x^2+x}$ $v^0 = 1 \Rightarrow v^{(1)} = 0$

$U^n = (2x+1)^n e^{x^2+x}$

$y = U^n \cdot V^0$

$y^n = (2x+1)^n e^{x^2+x} \cdot 1$

$y' = (2x+1)' e^{x^2+x}$

$V = 2x+1$, $U' = 2$ $V'' = 0$

$U^n = (2x+1)^n e^{x^2+x}$

$y'' = U^n V^0 + nU^{(n-1)} V' + 0$

$y'' = (2x+1)^1 e^{x^2+x} \cdot 2x+1$

$+ n(2x+1)^{(1-1)} e^{x^2+x} 2 + 0$

$y'' = (2x+1) e^{x^2+x} (2x+1) + 1 \cancel{(2x+1)}^0 e^{x^2+x} \cdot 2$

Recall $(2x+1)' e^{x^2+x} = y'$, $e^{x^2+x} = y$

$y'' = y'(2x+1) + 2y$

Prove that $y^{(n+2)} = (2x+1) y^{(n+1)}$

$+ 2(n+1) y^n$

where $w = y''$

$y'' - y'(2x+1) - 2y = 0$

$u = y''$, $U^n = y^{(n+2)}$, $V = 1$, $V' = 0$

$w^n = U^n \cdot V + 0$

$w^n = y^{(n+2)} \cdot 1$

$w^n = y^{(n+2)}$

where $w^n = (2x+1)y$

$u = y'$ $U^n = y^{(n+1)}$, $V = 2x+1$

$V' = 2$, $V'' = 0$

$w^n = U^n \cdot V^0 + nU^{(n-1)} V'$

$w^n = y^{(n+1)} (2x+1) + ny^n \cdot 2$

where $w^n = 2y$

$u = y$, $U^n = y^n$, $V = 2$, $V' = 0$

$w^n = U^n V^0$

$= y^n \cdot 2$

$= 2y^n$

$\therefore y^n = y^{(n+2)} - [y^{(n+1)}$

$(2x+1) + n2y^n] - [2y^n] = 0$

$y^{(n+2)} = y^{(n+1)}(2x+1) +$

$n2y^n + 2y^n$

$y^{(n+2)} = (2x+1) y^{(n+1)} + \cancel{2y^n}$

$2y^n(n+1)$

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2) $y = x^3 e^{4x}$ determine $y^{(5)}$

$u = e^{4x}$, $u^n = 4^n e^{4x}$

$v = x^3$, $v^1 = 3x^2$, $v^{(2)} = 6x$, $v^{(3)} = 6$, $v^{(4)} = 0$

$y^n = u^n \cdot v + n u^{(n-1)} \cdot v^1 + \frac{n(n-1) u^{(n-2)} \cdot v^{(2)}}{2!}$

$+ \frac{n(n-1)(n-2)}{3!} + u^{(n-3)} \cdot v^{(3)}$

$+ \frac{n(n-1)(n-2)(n-3)}{4!} u^{(n-4)} \cdot v^4$

$y^5 = 4^5 e^{4x} \cdot x^3 + 5 \cdot 4^{(5-1)} e^{4x} \cdot 3x^2 + \frac{5(5-1) 4^{(5-2)} e^{4x}}{2!} \cdot 6x + \cdots +$

$+ \frac{5(5-1)(5-2) 4^{(5-3)}}{3!} e^{4x} \cdot 6 + 0$

$y^5 = 4^5 e^{4x} \cdot x^3 + 5 \cdot 4^4 e^{4x} \cdot 3x^2$

$+ \frac{5 \cdot 4 \cdot 4^3}{2 \times 1} e^{4x} \cdot 6x + \frac{5(4)(3) 4^{(2)}}{3 \times 2 \times 1} e^{4x} \cdot 6$

$y^5 = 4^5 e^{4x} \cdot x^3 + 5 \cdot 4^4 e^{4x} \cdot 3x^2 + 5 \cdot 4^3$

$e^{4x} \cdot 6x + 5 \cdot 4 \cdot 3 \cdot 4^2 e^{4x}$

$y^5 = 1024 e^{4x} \cdot x^3 + 3840 e^{4x} \cdot x^2$

$+ 3840 e^{4x} \cdot x + 960 e^{4x}$

$y^5 = 64 e^{4x} (16x^3 + 60x^2 + 60x + 15)$

ii) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

show that

$x^2 y^{(n+2)} + (2n+1) x y^{(n+1)}$

$+ (n^2+1) y^n = 0$

$x^2 y^{11} + x y^1 + y = 0$

$w^n = x^2 y^{11}$

$u = y^{11}$, $u^n = y^{n+2}$, $v = x^2$

$v^1 = 2x$, $v^{11} = 2$, $v^{111} = 0$

$w^n = u^{(n)} v^0 + n u^{(n-1)} v^1 +$

$\frac{n(n-1) u^{(n-2)} v^{11}}{2!} +$

$\frac{n(n-1)(n-2) v^{(n-3)}}{3!}$

$v^{(n)}$

$w^n = y^{(n+2)} \cdot x^2 + n y^{(n+1)} \cdot 2x$

$+ \frac{n(n-1)}{2!} y^n \cdot 2 + 0$

$w^n = x^2 y^{n+2} + 2xny^{(n+1)}$

$+ n(n-1) y^n$

$\therefore x^2 y^{11} = x^2 y^{n+2} + 2xny^{(n+1)}$

$2xny^{(n+1)} + n(n-1) y^n$

$u = y' \; ! \; u^n = y^{n+1}, \; v = x, \; v' = 1, \; v'' = 0$

$w^n = y^{n+1} \cdot x + n y^n \cdot 1 + 0$

$w^n = x y^{(n+1)} + n y^n$

$\therefore \; x y' = x y^{(n+1)} + n y^n$

$u = y, \; u^n = y^n, \; v = 1, \; v' = 0$

$w^n = u^n v + n u^{(n-1)} \cdot v'$

$w^n = y^n$

$\therefore \; y = y^n$

$\therefore \; y^n = x^2 y^{(n+2)} + 2xny^{(n+1)}$

$+ n(n-1) y^n + x y^{(n+1)} + n y^n + y^n$

$y^n = x^2 y^{(n+2)} + 2xny^{(n+1)} + xy^{(n+1)}$

$+ n(n-1) y^n + \cdots n y^n + n$

$y^n = x^2 y^{(n+2)} + (xy + 2xn) y^{(n+1)}$

$+ (n(n-1) + n + 1] y^n$

$y^n = x^2 y^{(n+2)} + (xy + 2xn] y^{(n+1)}$

$+ (n(n-1) + n + 1) y^n$

$y^n = x^2 y^{(n+2)} + x y^{(n+1)} (2n+1)$

$+ (n^2 - n + n + 1] y^n$

$\therefore \; y^n = x^2 y^{(n+2)} + (2n+1)$

$x y^{(n+1)} + (n^2 + 1) y^n = 0$