FRENCH ERERE

16/ENG03/059

CIVIL ENGINEERING

ENG 381

1) If $y = e^{x^2} + x$

show that $y'' = y'(2x+1) + 2y$ and hence prove that $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

soln

$y' = (2x+1)e^{x^2+x}$

$y'' = (2x+1)(2x+1)e^{x^2+x} + 2e^{x^2+x}$

$\underbrace{y''}_{w_1} = \underbrace{y'(2x+1)}_{w_2} + \underbrace{2y}_{w_3}$

For w_1

y^{n+2}

for w_2

$u = y'$ $\quad v = 2x+1$

$u^{(n)} = y^{(n+1)}$ $\quad v' = 2$

$u^{(n-1)} = y^{(n)}$ $\quad v'' = 0$

$y^{(n+1)} 2x+1 + n y^{(n)} 2$

for w_3

$2y^{(0)}$

$2y^n$

$w_1 = w_2 + w_3$

$y^{(n+2)} = y^{(n+1)}(2x+1) + ny^{(n)} 2 + 2y^n$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2ny^{(n)} + 2y^n$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

2) $y = x^3 e^{4x}$ determine $y^{(5)}$

$u = e^{4x}$ $\qquad v = x^3$

$u^n = 4^n e^{4x}$ $\qquad v' = 3x^2$

$u^{(n-1)} = 4^{n-1} e^{4x}$ $\qquad v^2 = 6x$

$u^{(n-2)} = 4^{n-2} e^{4x}$ $\qquad v^3 = 6$

$u^{(n-3)} = 4^{n-3} e^{4x}$ $\qquad v^4 = 0$

$$y^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v^2 + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v^3$$

$$\cancel{+ \frac{n(n-1)(n-2)(n-3)}{4!} u^{(n-4)} v^4} \to 0$$

$$y^n = 4^n e^{4x} x^3 + n 4^{n-1} e^{4x} 3x^2 + \frac{n(n-1)}{2} 4^{n-2} e^{4x} 6x + \frac{n(n-1)(n-2)}{3!} 4^{n-3} e^{4x} 6$$

$$y^n = 4^n e^{4x} x^3 + n 4^{n-1} e^{4x} 3x^2 + n(n-1) 4^{n-2} e^{4x} 3x$$

$$y^5 = 4^5 e^{4x} x^3 + 5 \times 4^4 e^{4x} 3x^2 + 5(5-1) 4^3 e^{4x} 3x$$

$$y^5 = 1024 x^3 e^{4x} + 3840 x^2 e^{4x} + 3840 x e^{4x}$$

2ii) $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$ show that $x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^{(n)} = 0$

$$\underbrace{x^2 y^{(2)}}_{w_1} + \underbrace{x y^{(1)}}_{w_2} + \underbrace{y}_{w_3} = 0$$

for w_1

$u = y^{(2)}$ $\qquad v = x^2$

$u^{(n)} = y^{(n+2)}$ $\qquad v^{(1)} = 2x$

$u^{(n-1)} = y^{(n+1)}$ $\qquad v^{(2)} = 2$

$u^{(n-2)} = y^{(n)}$ $\qquad v^{(3)} = 0$

$$= y^{(n+2)} x^2 + n y^{(n+1)} 2x + \frac{n(n-1)}{2!} y^n 2 + 0$$

For ω_2

$u = y^{(1)}$ $\qquad$ $v = x$

$u^{(n)} = u^{(n+1)}$ $\qquad$ $v' = 1$

$u^{(n-1)} = y^{(n)}$ $\qquad$ $v'' = 0$

$= y^{(n+1)} x + n y^{(n)} \cdot 1 + 0$

for ω_3

$y^{(n)}$

$x^2 y^{(n+2)} + 2xny^{(n+1)} + n(n-1) y^n + xy^{n+1} + ny^{(n)} + y^{(n)} = 0$

$x^2 y^{(n+2)} + 2nxy^{(n+1)} + n(n+1) y^n + xy^{n+1} + ny^{(n)} + y^{(n)} = 0$

$x^2 y^{(n+2)} + (2n+1) xy^{(n+1)} + [n(n-1) + n + 1] y^{(n)} = 0$

$x^2 y^{(n+2)} + (2n+1) xy^{(n+1)} + [n^2 - n + n + 1] y^{(n)} = 0$

$x^2 y^{(n+2)} + (2n+1) xy^{(n+1)} + [n^2 + 1] y^{(n)} = 0$