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SUBJECT: ENG381

1) if $y = e^{x^2+x}$, show that $y = y(2x+1) + 2y$ and hence prove that $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

Answer

$u = e^{x^2+x}$; $V = 1$; $V' = 0$ $\quad y = e^{x^2+x}$ —— (i)

$y = e^{x^2+x}$; $u^n = (2x+1)^n e^{x^2+x}$

Using leibnitz theorem; $y^n = u^n V + n u^{(n-1)} V' + \frac{n(n-1)}{2!} u^{(n-2)} V^2$

$\Rightarrow (2x+1)^n e^{x^2+x} \cdot 1 + n(2x+1)^{(n-1)} e^{x^2+x} \cdot 0$

when $y^{(n)} \Rightarrow y^{(1)}$

$y' = (2x+1)' e^{x^2+x} \cdot 1$ —— (ii)

y'' = from y' ; $V = 2x+1$; $V' = 2$; $V'' = 0$

$u^n = (2x+1)^n e^{x^2+x}$

$y'' = (2x+1)' e^{x^2+x} \cdot 2x+1 + n(2x+1)^{1-1} e^{x^2+x} \cdot 2 + 0$

$y'' = (2x+1)' e^{x^2+x} \cdot (2x+1) + 1(2x+1)^0 e^{x^2+x} \cdot 2$

Substitute equ (2) into (ii)

$y'' = y'(2x+1) + 2y$

1b) Prove that $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

answer

Recall that $y'' = y'(2x+1) + 2y$, $y'' - y'(2x+1) - 2y = 0$

for $y'' \Rightarrow$ $\cancel{u} = y''$

$u = y''$; $u^n = y^{(n+2)}$

$V = 1$, $V' = 0$

$y^n = u^n \cdot V + 0$

$\Rightarrow y^{(n+2)} \cdot 0$

for $(y'(2x+1))$ $\Rightarrow$ $y^n = y'(2x+1)$; $u = y'$; $u^n = y^{(n+1)}$, $V = 2x+1$; $V' = 2$; $V'' = 0$

$y^n = u^n V + n u^{(n-1)} V' + 0$

$\Rightarrow y^{(n+1)}(2x+1) + n y^n \cdot 2$

for $2y$; $w = 2y$; $u = y$; $u^n = y^n$

$V = 2$; $v' = 0$

$y^n = u^n . v + 0$

$y^n = \cancel{y}y^n . 2$

$y^n = 2y^n$

$\therefore y^n = y^{(n+2)} - (y^{(n+1)}(2x+1) + n2y^n) - 2y^n = 0$

$y^{(n+2)} = y^{(n+1)}(2x+1) + n2y^n + 2y^n$

$\therefore y^{(n+2)} = (2x+1)\, y^{(n+1)} + 2y^n (n+1)$

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2) Using leibnitz theorem, given that

1) $y = x^3 e^{4x}$, determine $y^{(5)}$

<u>Answer</u>

$w = x^3 e^{4x}$

$x^3 \to v$, $e^{4x} \to u$

$v = x^3$; $v' = 3x^2$; $v'' = 6x$; $v''' = 6$; $v^{iv} = 0$

$u = e^{4x}$; $u^n = 4^n e^{4x}$

Using leibnitz theorem

$$y^n = u^n v + n u^{(n-1)} v^1 + \frac{n(n-1)}{2!} u^{(n-2)} v^2 + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v^3 + \dots$$

when $y^{(5)}$

$$y^{(n)} = 4^n e^{4x} \cdot x^3 + n 4^{(n-1)} e^{4x} \cdot 3x^2 + \frac{n(n-1)}{2!} 4^{(n-2)} e^{4x} \cdot 6x + \frac{n(n-1)(n-2)}{3!}$$

$$4^{(n-3)} e^{4x} \cdot 6 + \frac{n(n-1)(n-2)(n-3)}{4!} 4^{(n-4)} e^{4x} \cdot 0$$

$$y^{(5)} = 4^{(5)} e^{4x} \cdot x^3 + (5) 4^{(5-1)} e^{4x} \cdot 3x^2 + \frac{5(5-1)}{2!} 4^{(5-2)} e^{4x} \cdot 6x +$$

$$\frac{5(5-1)(5-2)}{3!} 4^{(5-3)} e^{4x} \cdot 6 + 0$$

$$\Rightarrow 1024 e^{4x} x^3 + 1280 e^{4x} \cdot 3x^2 + \frac{5(4)}{2 \times 1} 64 \cdot e^{4x} \cdot 6x + \frac{5(4)(3)}{3 \times 2 \times 1} 16$$

$$\cdot e^{4x} \cdot 6 + 0$$

$$\Rightarrow 1024 x^3 e^{4x} + 3840 x^2 e^{4x} + 3840 x e^{4x} + 480 e^{4x} + 0$$

$$\therefore y^{(5)} \Rightarrow 1024 x^3 e^{4x} + 3840 x^2 e^{4x} + 3840 x e^{4x} + 480 e^{4x}$$

B) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$, show that $x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^{(n)} = 0$

<u>Answer</u>

$x^2 y'' + x y' + y = 0$ [Homogenous second derivative]

$\boxed{\text{for } x^2 y''}$; $w = uv$ where $v = x^2$; $v' = 2x$; $v'' = 2$; $v''' = 0$

$u = y''$; $u^n = y^{(n+2)}$

Using Leibnitz theorem

$$y^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v^2 + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} \dots$$

$$\Rightarrow y^{(n+2)} \cdot x^2 + n y^{(n+2-1)} \cdot 2x + \frac{n(n-1)}{2!} y^{(n+2-2)} \cdot 2 + \cancel{\frac{n(n-1)(n-2)}{3!} y^{(n+2-3)} \cdot 0}$$

$$\Rightarrow x^2 y^{(n+2)} + 2xn y^{(n+1)} + \frac{n(n-1)}{2 \times 1} y^n \cdot 2$$

$x^2 y''$ $\Rightarrow x^2 y^{(n+2)} + 2xny^{(n+1)} + n(n-1) y^n$

$\boxed{\text{for } xy'}$: $w = uv$ where $v = x$; $v' = 1$; $v'' = 0$

$u = y'$; $u^n = y^{(n+1)}$

Using Leibnitz theorem

$$y^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v^2 + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v^3 + \dots$$

$$y^n = y^{(n+1)} x + n y^{(n+1-1)} \cdot 1 + \frac{n(n-1)}{2!} y^{(n+1-2)} \cdot 0$$

$$y^n = xy^{(n+1)} + ny^{(n)} + 0$$

for xy' : $y^n = xy^{(n+1)} + ny^{(n)}$ //

for y ; $w = uv$ where $v = 1$; $v' = 0$

$u = y$; $u^n = y^n$

Using Leibnitz theorem,

$$\Rightarrow y^n \cdot 1 + ny^{(n-1)} \cdot 0$$

$$\Rightarrow y^n$$

$\therefore$ combining all solutions

$$x^2 y'' + xy' + y = \left[x^2 y^{(n+2)} + 2xny^{(n+1)} + n(n-1) y^n\right] + \left[xy^{(n+1)} + ny^{(n)}\right] + \left[y^n\right]$$

Taking common factors

$$\Rightarrow x^2 y^{(n+2)} + xy^{(n+1)}[2n+1] + y^n[n(n-1) + n + 1]$$

$$\Rightarrow x^2 y^{(n+2)} + xy^{(n+1)}[2n+1] + y^n[n^2 - n + n + 1]$$

$$\Rightarrow x^2 y^{(n+2)} + xy^{(n+1)}[2n+1] + y^n[n^2+1]$$

$$\therefore x^2 y'' + xy' + y = x^2 y^{(n+2)} + xy^{(n+1)}[2n+1] + y^n[n^2+1] = 0$$