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15/ENG06/050
ENG 381

ASSIGNMENT 3

1. If $y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$

$y'' = y'(2x+1) + 2y$

$y' = (2x+1)e^{x^2+x}$

$y' \cdot (2x+1) = (2x+1)e^{x^2+x}$

$= (2x+1)^2 e^{x^2+x}$

$2y = 2 \cdot e^{x^2+x}$

$= y'(2x+1) + 2y = (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$

$\therefore y'' = y'(2x+1) + 2y$

$\therefore 2e^{x^2+x} + (2x+1)^2 e^{x^2+x} = (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$

$w = y''$

$w^n = y^{n+2}$

$P = y'(2x+1)$

$y = 2x+1$

$v' = 2$ $\qquad u = y'$

$v'' = 0$ $\qquad u^n = y^{n+1}$

$P^n = y^{n+1} \cdot 2x+1 + n \cdot y^n \cdot 2$

$S = 2y$

$S^n = 2y^n$

$w^n = P^n + S^n$

$y^{n+2} = (2x+1)y^{n+1} + 2ny^n + 2y^n$

$y^{n+2} = (2x+1)y^{n+1} + 2(n+1)y^n$

2. Using the Leibnitz theorem, given that $y = x^3 e^{4x}$ determine y^5

$v = x^3$ $\qquad u = e^{4x}$

$v' = 3x^2$ $\qquad u' = 4e^{4x}$

$v'' = 6x$ $\qquad u'' = 4 \cdot 4e^{4x}$

$v''' = 6$ $\qquad u''' = 4 \cdot 4 \cdot 4e^{4x}$

$v^{iv} = 0$ $\qquad u^n = 4^n \cdot e^{4x}$

$$y^n = 4^n e^{4x} \cdot x^3 + n \cdot 4^{n-1} e^{4x} \cdot 3x^2 + \frac{n(n-1)}{2} 4^{n-2} e^{4x} \cdot 6x +$$

$$\frac{n(n-1)(n-2)}{6} 4^{n-3} e^{4x} \cdot 6$$

$$y^n = 4^n e^{4x} x^3 + n 4^{n-1} e^{4x} 3x^2 + n(n-1) 4^{n-2} e^{4x} 3x +$$
$$n(n-1)(n-2) 4^{n-3} e^{4x}$$

$$y^5 = 4^5 e^{4x} x^3 + 4e^{4x} 3x^2 + 5 \cdot 4 \cdot 4^3 e^{4x} 3x + 5 \cdot 4 \cdot 3 \cdot 4^2 \cdot e^{4x}$$

Q6.

$x^2 y'' + xy' + y = 0$ Show that $x^2 y^{(n+2)} + (2n+1) xy^{(n+2)} + (n^2+1) y^n = 0$

let $w = x^2 y''$

$V = x^2$ $\qquad u = y''$

$V' = 2x$ $\qquad u' = y'''$

$V'' = 2$ $\qquad u' = y^{(iv)}$

$V''' = 0$ $\qquad u^n = y^{n+2}$

$$w^n = y^{n+2} x^2 + n \cdot y^{n+1} 2x + \frac{n(n-1)}{2} \cdot y^n \cdot 2$$

$$w^n = x^2 y^{n+2} + n 2x y^{n+1} + n(n-1) y^n$$

let $P = xy'$

$V = x$ $\qquad u = y'$

$V' = 1$ $\qquad u' = y''$

$V'' = 0$ $\qquad u^n = y^{n+1}$

$$p^n = y^{n+1} x + n \cdot y^n \cdot 1$$
$$p^n = x y^{n+1} + n y^n$$

$S = y$

$S^n = y^n$

$$y^{n+2} \cdot x^2 + n \cdot 2xy^{n+1} + n(n-1)\, y^n + y^{n+1} x + ny^n + y^n = 0$$

$$y^{n+2} x^2 + (n \cdot 2x\, y^{n+1} + x\, y^{n+1}) + (n(n-1)\, y^n + ny^n + y^n) = 0$$

$$x^2 y^{n+2} + (2n+1)\, xy^{n+1} + (n(n-1) + 1 + n)\, y^n = 0$$

$$x^2 y^{n+2} + (2n+1)\, xy^{n+1} + (n^2+1)\, y^n = 0$$