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ASSIGNMENT SOLUTIONS.

1. If $y = e^{x^2+x}$

$u = e^{x^2+x}$ $\quad v^0 = 1 \Rightarrow v^{(1)} = 0$.

$u^n = (2x+1)^n e^{x^2+x}$

$y = u^n . v^0$

$y^n = (2x+1)^n e^{x^2+x} . 1$

$y' = (2x+1)' e^{x^2+x}$

$v = 2x+1, \quad v' = 2, \quad v'' = 0$

$u^n = (2x+1)^n e^{x^2+x}$

$y'' = u^n v^0 + n u^{(n-1)} v' + 0$

$y'' = (2x+1)' e^{x^2+x} . 2x+1 + n(2x+1)^{(1-1)} e^{x^2+x} . 2 + 0$

$y'' = (2x+1) e^{x^2+x} (2x+1) + 1\,(2x+1)^0 e^{x^2+x} . 2$

Recall $(2x+1)' e^{x^2+x} = y'$, $\quad e^{x^2+x} = y$.

$y'' = y'(2x+1) + 2y$.

Prove that $y^{(n+2)} = (2x+1) y^{(n+1)} + 2(n+1) y^n$

where $w = y''$

$y'' - y'(2x+1) - 2y = 0$

$u = y''$, $\quad u^n = y^{(n+2)}$, $\quad v = 1$, $\quad v' = 0$

$w^n = u^n . v + 0$

$w^n = y^{(n+2)} . 1$

$W_n = y^{(n+2)}$

where $W^n = (2x+1)y$

$u = y'$ $u^n = y^{(n+1)}$, $v = 2x+1$, $v' = 2$, $v'' = 0$

$W^n = u^n \cdot v^0 + n u^{(n-1)} v'$

$W^n = y^{(n+1)}(2x+1) + n y^n \cdot 2$

where $W^n = 2y$

$u = y$, $u^n = y^n$, $v = 2$, $v' = 0$

$W^n = u^n v^0$

$= y^n \cdot 2$

$= 2y^n$.

$\therefore y^n = y^{(n+2)} - [y^{(n+1)}(2x+1) + n2y^n] - [2y^n] = 0$

$y^{(n+2)} = y^{(n+1)}(2x+1) + n2y^n + 2y^n$

$y^{(n+2)} = \underline{(2x+1)y^{(n+1)}} + 2y^n(n+1)$

2. i. $y = x^3 e^{4x}$ determine $y^{(5)}$

$u = e^{4x}$, $u^n = 4^n e^{4x}$

$v = x^3$, $v' = 3x^2$, $v^{(2)} = 6x$, $v^{(3)} = 6$, $v^{(4)} = 0$

$y^n = u^n \cdot v + n u^{(n-1)} \cdot v' + \frac{n(n-1)u^{(n-2)} \cdot v^{(2)}}{2!} + \frac{n(n-1)(n-2)}{3!} + \ldots$

$+ u^{(n-3)} \cdot v^{(3)} + \frac{n \cdot (n-1)(n-2)(n-3)}{4!} u^{(n-4)} \cdot v^4$

$$y^5 = 4^5 e^{4x} \cdot x^3 + 5 \cdot 4^{(5-1)} e^{4x} \cdot 3x^2 + \frac{5(5-1)\, 4^{(5-2)} e^{4x}}{2!} \cdot 6x + \cdots$$

$$+ \frac{5(5-1)(5-2)\, 4^{(5-3)} e^{4x} \cdot 6}{3!} + 0$$

$$y^5 = 4^5 e^{4x} \cdot x^3 + 5 \cdot 4^4 e^{4x} \cdot 3x^2 + \frac{5\cancel{4}\, \cancel{4}^3 e^{4x} \cdot 6x}{\cancel{2 \times 1}} + \frac{5^{(4)(3)} 4^{(2)} e^{4x} \cdot 6}{\cancel{3 \times 2 \times 1}}$$

$$y^5 = 4^5 e^{4x} \cdot x^3 + 5 \cdot 4^4 e^{4x} \cdot 3x^2 + 5 \cdot 2 \cdot 4^3 e^{4x} \cdot 6x + 5 \cdot 4 \cdot 3 \cdot 4^2 e^{4x}$$

$$y^5 = 1024 e^{4x} x^3 + 3840 e^{4x} \cdot x^2 + 3840 e^{4x} \cdot x + 960 e^{4x}.$$

$$y^5 = 64 e^{4x} [16x^3 + 60x^2 + 60x + 15].$$

ii. $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$. Show that

$$x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^n = 0$$

$$x^2 y'' + x y' + y = 0$$

$$w^n = x^2 y''$$

$$u = y'', \quad u^n = y^{n+2}, \quad v = x^2, \quad v' = 2x, \quad v'' = 2, \quad v''' = 0$$

$$w^n = u^{(n)} v^0 + n u^{(n-1)} v' + \frac{n(n-1)\, u^{(n-2)} v''}{2!} + \frac{n(n-1)(n-2)\, u^{(n-3)}}{3!}$$

v'''.

$$w^n = y^{(n+2)} \cdot x^2 + n \cdot y^{(n+1)} \cdot 2x + \frac{n(n-1)\, y^n \cdot \cancel{2}}{\cancel{2}!} + 0$$

$$w^n = x^2 y^{n+2} + 2xn\, y^{(n+1)} + n(n-1) y^n.$$

$$\therefore x^2 y'' = x^2 y^{n+2} + 2xn\, y^{(n+1)} + n(n-1) y^n$$

$u = y'$, $u^n = y^{n+1}$, $v = x$; $v' = 1$, $v'' = 0$

$w^n = y^{n+1} \cdot x + n y^{n-1} + 0$

$w^n = xy^{(n+1)} + ny^n$.

$\therefore xy' = xy^{(n+1)} + ny^n$.

$u = y$, $u^n = y^n$, $v = 1$ $v' = 0$

$w^n = u^n v + n u^{(n-1)} \cdot v'$

$w^n = y^n$.

$\therefore y = y^n$.

$\therefore y^n = x^2 y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^n + xy^{(n+1)} + \ldots$
$+ ny^n + y^n$

$y^n = x^2 y^{(n+2)} + 2xny^{(n+1)} + xy^{(n+1)} + n(n-1)y^n + \ldots$
$ny^n + ^n$

$y^n = x^2 y^{(n+2)} + (xy + 2xn) y^{(n+1)} + [n(n-1) + n + 1] y^n$

$y^n = x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + (n^2 - n + n + 1] y^n$

$\therefore y^n = x^2 y^{(n+2)} + (2n+1)\, xy^{(n+1)} + (n^2+1) y^n = 0.$