2(ii)

$x^2y'' + xy' + y = 0$

SUB 1:

$W = x^2y''$

$V = x^2 \quad V' = 2x, \; V'' = 2, \; V''' = 0$

$U = y'' \quad U^n = y^{(n+2)}$

$W^{(n)} = y^{(n+2)} \cdot x^2 + ny^{n+1} \cdot 2x + \frac{n(n-1)}{2} y^n \cdot 2 + \frac{n(n-1)(n-2)}{6} y^{n-1} + 0$

$= y^{(n+2)} x^2 + 2nxy^{(n+1)} + n(n-1)y^n$

SUB 2:

$W = xy'$

$V = x \quad V' = 1 \quad V'' = 0$

$U = y' \quad U^n = y^{(n+1)}$

$W^{(n)} = y^{(n+1)} x + ny^n$

SUB 3:

$W = y$

$V = 1 \quad V' = 0$

$U = y \quad U^n = y^n$

$W^{(n)} = y^n$

Then $x^2y'' + xy' + y = 0$ becomes

$y^{(n)} = x^2 y^{(n+2)} + 2nxy^{(n+1)} + n(n-1)y^{(n)} + xy^{(n+1)} + ny^n + y^n$

$= x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + y^{(n)} (n^2+1) = 0$

2(i)

$$y = x^3 \cdot e^{4x} \quad \text{Find } y^{(5)}.$$

$$u = e^{4x} : u^{(n)} = 4^n e^{4x}$$

$$v = x^3 : v' = 3x^2,\ v'' = 6x,\ v''' = 6,\ v^{(4)} = 0$$

$$y^{(n)} = u^{(n)} \cdot v + n u^{(n-1)} v^{(1)} + \frac{n(n-1)}{2!} u^{(n-2)} v^{(2)} + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v^{(3)} + \frac{n(n-1)(n-2)(n-3)}{4!} u^{(n-4)} v^{(4)} + \frac{n(n-1)(n-2)(n-3)(n-4)}{5!} u^{(n-5)} v^{(5)} + \dots$$

$$y^{(5)} = 4^5 e^{4x} \cdot x^3 + 5 \cdot 4^4 e^{4x} \cdot 3x^2 + \frac{5(4)}{2} 4^3 e^{4x} \cdot 6x + \frac{5(4)(3)}{6} 4^2 e^{4x} \cdot 6 + 0$$

$$y^{(5)} = 1024 e^{4x} x^3 + 3840 e^{4x} x^2 + 3840 e^{4x} \cdot x + 960 e^{4x}$$

$$y^{(5)} = 64 e^{4x}(16x^3 + 60x^2 + 60x + 15)$$
$$= 4^3 e^{4x}(16x^3 + 60x^2 + 60x + 15)$$

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