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15/ENG07/008

CHEMICAL ENGINEERING

ENG 381: ENGINEERING MATHEMATICS 3

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ASS

2) Using the Leibnitz theorem, given that
i) $y = x^3 e^{4x}$ determine $y^{(5)}$

Sol

Recall the Leibnitz theorem

$$y = u^5 v + 5u^4 v' + 10u^3 v^2 + 10u^2 v^3 + 5u' v^4 + v^5$$

Where
$u = e^{4x}$ $\quad v = x^3$
$u' = 4e^{4x}$ $\quad v' = 3x^2$
$u^2 = 16e^{4x}$ $\quad v^2 = 6x$
$u^3 = 64e^{4x}$ $\quad v^3 = 6$
$u^4 = 256e^{4x}$
$u^5 = 1024e^{4x}$

Hence

$$y^{(5)} = 1024e^{4x}(x^3) + 5(256e^{4x})(3x^2) + 10(64e^{4x})(6x) + 10(16e^{4x})(6) + 5(4e^{4x})(0) + (0)^5$$

$$y^{(5)} = 1024x^3 \cdot e^{4x} + 3840x^2 \cdot e^{4x} + 3840x \cdot e^{4x} + 960e^{4x}$$

$$y^{(5)} = e^{4x}(1024x^3 + 3840x^2 + 3840x + 960)$$

ii) If $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$, show that $x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$.

Sol

The Equation can be rewritten as $x^2 y'' + xy' + y = 0$

Let $w_1 = x^2 y''$, Let $xy' = w_2$, Let $w_3 = y$

For w_1:
$u = y^{(2)}$ $\quad v = x^2$ $\quad v'' = 2$
$u^{(n)} = y^{(n+2)}$ $\quad v' = 2x$ $\quad v''' = 0$

$u^{(n-1)} = y^{(n-1+2)} = y^{(n+1)}$

$u^{(n-2)} = y^{(n)}$

$w_1 = y^{(n+2)} \cdot x^2 + n y^{(n+1)} 2x + \frac{n(n-1)}{2!} y^n \cdot 2 + 0$

$w_2 = xy'$

$w_2 = y^{(n+1)} \cdot x + n y^{(n)} \times 1 + 0$

$w_3 = y^{(n)}$

$w_1 + w_2 + w_3$

$x^2 y^{(n+2)} + n y^{(n+1)} \cdot 2x + \frac{n(n-1)}{2} \times 2 \times y^n + xy' + y^{(n+1)} \cdot x$

$+ n y^{(n)} + y^{(n)} = 0$

$x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + y^{(n)}(n(n-1) + n + 1) = 0$

$x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2 - \cancel{n} + n + 1) y^n = 0$

$x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^{(n)} = 0$

QED

1) If $y = e^{x^2 + x}$, show that $y'' = y'(2x+1) + 2y$ and hence prove that $y^{(n+2)} = (2x+1) y^{(n+1)} + 2(n+1) y^n$.

Sol

$y = e^{x^2 + x}$

Let $u = x^2 + x$

$\frac{du}{dx} = 2x + 1$

$y = e^u$

$\frac{dy}{du} = e^u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$\frac{dy}{dx} = e^u \times 2x + 1$

where $u = x^2 + x$

$\frac{dy}{dx} = e^{x^2 + x} \cdot (2x+1) = y'$

if $y' = e^{x^2+x}(2x+1)$

Then

$y'' = 2e^{x^2} e^{x^2+x}(2) + \left[e^{x^2+x}\cdot(2x+1)\right](2x+1$

$$y'' = 2\cdot e^{x^2+x} + e^{x^2+x}\cdot(2x+1)\cdot(2x+1)$$

but $y = e^{x^2+x}$

and $y' = e^{x^2+x}(2x+1)$

$\therefore$

$$y'' = 2(y) + y'(2x+1) \qquad \text{Q 2D}$$

Applying Leibnitz theorem to the above Equation

finding its nth derivative

$$y^{(n+2)} = 2(n+1)y^{n} + y^{(n+1)}(2x+1)$$

$$=$$